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Precocious Scaling and Duality in the Quark-Parton Model--A Reformulation

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ABSTRACT

In a model of confinement similar to the Massive Quark Model, we take up the subjects of precocious scaling and the Bloom-Gilman relation. We identify precocious scaling with exchange degeneracy among secondary trajectories. By using the Annihilation Rule, which explains the pattern of exchange degeneracy in hadron reactions, we show that $e^- N \to e^- X$ should manifest precocious scaling, as should some other processes such as $e^- e^+ \to \pi^{\pm} X$, $e^- e^+ \to KX$. In contrast thereto, certain other reactions, such as $e^- e^+ \to h^0 X$, which allow an isosinglet missing mass, may not show precocious scaling. Examples include $h^0 = \pi^0, \eta, \omega, \phi$. The predictions of precocious scaling are only as good as the valence quark model and hence, only apply for x sufficiently near to one. For x sufficiently far from one, precocious scaling will not be valid for any reaction. Another consequence is that the total cross section for electron-positron annihilation to hadrons need not scale rapidly. However, in the limit of SU_3 invariance, even this cross section would scale precociously.

In our picture, the Bloom-Gilman relation corresponds to having resonances in three quark scattering dual to the pomeron. This relation is shown to be consistent with a generalization of the dual pomeron to quark-quark and quark-antiquark scattering.

INTRODUCTION

In the following, we discuss the quark model, as applied to the processes of deep inelastic lepton scattering and electron-positron annihilation. The language used in the formal derivation will resemble the formulation of the covariant parton model,[1] although some of our assumptions differ therefrom and the basic physical picture is quite different. Our heuristic presentation, however, more closely resembles Feynman's discussion.[2] Although developed independently, the kinematical formalism is essentially the same as the Massive Quark Model (MQM).[3] However, we stress the possibility of dynamical confinement rather than infinitely massive quarks. The purpose of this paper is to suggest several logical alternatives to conventional parton models and, consequently, to extend the possibilities beyond those normally considered. In particular, our interpretation of precocious scaling and our formulation of duality are quite different from other treatments (including the MQM).

In Sec. I, we review the kinematics of deep inelastic lepton scattering and describe the analogy with single particle inclusive reactions. In Sec. II, we suggest that precocious scaling is a consequence of exchange degeneracy and, consequently, may be both particle dependent and x dependent. In Sec. III, we extend our understanding of duality in hadron reactions to electroproduction, introducing the Annihilation Rule as the basis for exchange degeneracy. This leads us to suggest that precocious scaling, in general, is only as good as the valence quark approximation and, hence, should break down, for all hadrons, the further x is from one. Secondly, we show that, within the valence quark approximation, $e^-e^+ \to \pi^\pm X$ will manifest precocious scaling, but $e^-e^+ \to \pi^0 X$ may not, even for x near one. We discuss other related processes as well. In Sec. IV A, we take up the total cross section σ_{tot}^{-+} for annihilation into hadrons and show that (a) the asymptotic behavior differs in normalization from the free quark model and (b) precocious scaling holds in limit of SU(3) symmetry. Somewhat dismayed by this, we entertain some speculations in Sec. IV B on how the quark model might be modified to give enhancement of σ_{tot}^{-+} without altering scaling in deep inelastic electroproduction or in $e^-e^+ \to hX$. Section V summarizes these formal results and outlines directions for future work. Section VI, indicates how the purely kinematic analogy between electroproduction and single particle inclusive reactions, suggests that one use rapidity y to study region near $x = 0$ and examine scaling in limit $Q^2 \to \infty$ for fixed $Q^2/\sqrt{\nu}$. In a lengthy Appendix A, we give a more detailed mathematical treatment of the model and derive the results presented in Sec. I. Nearly all of this is already contained in discussions of the MQM,[3] however, we emphasize some possibly fundamental inconsistencies in the present formulation. In Appendix B, we consider rules for determining the powers associated with quark and diquark exchange.

I. KINEMATICS AND SUMMARY OF DYNAMICAL ASSUMPTIONS

A more precise mathematical discussion is given in the Appendix. In this section, we will summarize the results obtained there. Our description of the amplitude for deep inelastic lepton scattering from a hadron h, $\ell^- h \rightarrow \ell^- X$, is depicted in Fig. 1. The dominant contribution comes from single photon exchange. We assume that, for moderate values of the photon invariant mass, its electromagnetic interaction with the hadron is mediated by a pointlike coupling with quarks. We assume also that no quarks are produced in the laboratory, so that only hadrons make up the missing mass X. Consequently, both quarks must interact with the hadron. To describe the nature of the interaction, it is useful to work in a Lorenz frame previously employed by Feynman.[2] We write

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\begin{aligned}
p &= \left(P + \frac{M^2}{2P}, \underset{\sim}{0}, P\right) \\
q &= (0, \underset{\sim}{0}, -2xP) \qquad\qquad (1) \\
\nu m &= p \cdot q = 2xP^2, \; Q^2 = 4x^2 P^2
\end{aligned}
$$

We assume that the quarks behave as if they have finite mass. We may think of a quark (with momentum k_j) colliding with the hadron (with momentum p) producing an outgoing quark (with momentum $k_j - q$) plus additional unobserved hadrons X. We parametrize the quark momentum as

$$k_j = (E_j, \underset{\sim}{K}_j, -x_j P) \quad . \tag{2}$$

For large ν, one can show that

$$x_j = x + \frac{\mu_j^2 - m_j^2}{2\nu m} \tag{3}$$

where $m_j(\mu_j)$ is the mass of the incoming (outgoing) quark. We further assume that the dominant contribution to the scattering comes from finite values of the transverse momentum $\underset{\sim}{K}_j$. We are interested in the asymptotic behavior of the amplitude for quark + hadron → quark + anything in the Bjorken limit: $\nu \to \infty$ for fixed x. This corresponds to the high energy limit in which the momentum transfer t_j between the hadron and outgoing quark remains finite, while the momentum transfer between quarks $u_j = Q^2$ increases proportional to the initial energy-squared $s_j \approx 2m\nu$. Thus the Bjorken limit is the Mueller-Regge limit of $k_1 + p \to (k_1 - q) + X$ corresponding to the fragmentation of the hadron into a quark. This is most easily described in terms of the discontinuity of the three-to-three amplitude (Fig. 2). Consequently, in the Bjorken limit, the asymptotic behavior is given by Regge singularities in the quark-antiquark channel, $t_{12} = (k_1 - k_2)^2 \approx -(\underset{\sim}{K}_1 - \underset{\sim}{K}_2)^2$ (see Fig. 3). If we identify the leading singularity with the pomeron with intercept one, then we find that, up to possible logarithmic corrections, the structure function νW_2 scales (see Appendix A),

$$\nu W_2(\nu, q^2) \xrightarrow[\text{fixed } x]{\nu \to \infty} \sum_i e_i^2 F_2^i(x) , \tag{4}$$

(sum over different quarks). For each type of quark, there is an additional contribution from the process antiquark + hadron → antiquark + anything. Accordingly, $F_2^i(x)$ is to be associated with limiting fragmentation of the hadron into quarks, and Bjorken scaling in deep inelastic scattering is related to Feynman's scaling in inclusive hadronic reactions. We can pursue this hadronic analogy to discuss the behaviors as $x \to 0$ and $x \to 1$. As $x \to 0$, the momentum transfer t_j grows, and the fragmentation function is determined by the Regge singularities in the hadron-antihadron channel (Fig. 4). This implies that (see Appendix A)

$$F_2(x) \xrightarrow{x \to 0} C_{\mathbb{P}} x^{1-\alpha_{\mathbb{P}}} + C_R x^{1-\alpha_R} \quad . \tag{5}$$

With $\alpha_P = 1$ and $\alpha_R \simeq \frac{1}{2}$, we get (up to logarithmic corrections)

$$F_2(x) \to C_{\mathbb{P}} + C_R \sqrt{x} \quad . \tag{6}$$

We see explicitly the relationship to the central plateau of hadron pionization. In Feynman's picture of the hadronic bremsstrahlung of a noninteracting quark, the development of the central plateau has remained a subject of controversy and continual discussion. One beauty of the present approach in which both quarks interact is the very natural way in which this plateau arises. This leads us to suggest, in Sec. VI, ways to test for its presence other than the behavior of $F_2(x)$ as $x \to 0$.

In hadronic reactions, the behavior of the single-particle fragmentation as $x \to 1$ is called the triple-Regge limit. The point is that as $x \to 1$, the dominant contribution comes from exchanges in the hadron-antiquark (or hadron-quark) channel (see Fig. 5). Consequently (up to possible logarithmic corrections), we find

$$F_2(x) \to (1-x)^{1-2\alpha} \tag{7}$$

where α is the intercept of the leading singularity in the hadron-antiquark channel. (For the pion, this corresponds to quark exchange; for the proton, to diquark exchange.)

Among the states contributing to the missing mass in Fig. 1 is the hadron pole itself. Consequently, the natural model for the large Q^2 behavior of the form factor corresponds (for spacelike q) to the asymptotic behavior of quark-hadron elastic scattering in the backward direction. (See Fig. 6 and Appendix A.) We then find

$$F_h(Q^2) \xrightarrow{Q^2 \to \infty} (Q^2)^{\alpha-1} \tag{8}$$

This relation between the asymptotic behavior of the form factor and the behavior of $F_2(x)$ as $x \to 1$ [Eq. (7)] is called the Drell-Yan-West relation.[4] Here it arises beautifully from a connection between the Regge asymptotic behavior of form factors and the helicity pole (triple Regge) limit of inclusive distributions.

In the case of spin, the situation is more complicated and still under investigation. It appears, however, that the Drell-Yan-West relation may be modified, in general, so that,

$$F_2(x) \to (1-x)^{n-2\alpha}$$

where n may be an integer different from 1.

We regard the powers α as not given by the model. Their association with elementary quark exchange is very tantalizing and is explored in Appendix B.

The approach to the scaling limit is also provided for in this model. In general, in addition to the pomeron (Fig. 3), there will be a sum over different exchanges α_a. If k_1 and k_2 represent the same type of quark (diagonal terms), the next to leading singularities will be f^0, ρ, ω, A_2 with intercepts $\alpha_a \approx 1/2$. In addition, there will be nondiagonal terms (where k_1 and k_2 are different quarks) to which charged ρ, A_2, as well as K^*, K^{**}, contribute. (Figs. 7a and b.) Also, there will be contributions coming from the u_{12} channel (Fig. 7c) requiring diquark exchange. We will assume that diquark has intercept less than zero and can be neglected compared to the Reggeons aforementioned. In summary, then, we expect scaling to be approached as $\nu^{-1/2}$.

This completes a synopsis of the basis framework and results of the model. Nearly all the preceding has been previously developed in the MQM.[3]

II. DUALITY: PRECOCIOUS SCALING AND THE BLOOM-GILMAN RELATION

At SLAC, scaling seems to be established[5] already at fairly small values of q^2,* typically for $|q^2| > 1$ or $2\ \text{GeV}^2$. This "precocious scaling" seems to contradict the $\nu^{-1/2}$ approach just derived; however, we recall that, in hadron scattering, there is precedence for the absence of leading secondary exchanges. That phenomenon is called exchange degeneracy and is the reason that the pp and K^+p total cross sections are more or less constant (compared to other channels) already at low energies. We are led to suggest that the observation of precocious scaling at SLAC is due to the cancellation among Reggeon exchanges, the leading ones of which are the f^0, ρ, ω and A_2. Just as in hadronic collisions, we would not necessarily expect precocious scaling to be a property of all hadrons.

Unfortunately, it is not experimentally feasible to perform deep inelastic scattering from meson targets.† However, a similar analysis seems to hold for electron-positron annihilation. Consequently, while $e^-e^+ \to hX$ should eventually scale, we might find scaling is precocious for some hadrons h but not for others. It may also happen that the phenomenon of rapid scaling is dependent on the value of x. In the next

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*We postpone to later, a detailed analysis of small deviations observed in Ref. 5.

†As G. Kane reminded us, there is the possibility of extrapolating certain reactions to their pion pole to obtain these indirectly.

section, we will take up a specific dual model and find both these possibilities realized.

According to Bloom and Gilman,[6] the nucleon resonances in deep inelastic electron scattering contribute to the scaling function, in some average sense. This corresponds to the situation described in Fig. 8, where we've assumed the dominant contribution for x near one comes from diquark exchange. The "average sense" referred to above can be made somewhat more precise, since the connection between finite missing mass and the asymptotic behavior is beautifully provided* for in so-called finite mass sum rules.[7] However, at first sight, the Bloom-Gilman (BG) relation seems odd, since the scaling function comes from pomeron exchange. Consequently, the translation of the BG hypothesis into our language is that, <u>in three quark scattering, the hadron resonances are dual to the pomeron.</u> (Presumably, hadronic background also contributes to the pomeron.) Ordinarily, we are used to thinking of resonances building ordinary Regge trajectories, but we have suggested above that precocious scaling corresponds to exchange degeneracy among secondaries, i.e., there should be no contribution from f^0, ω, ρ, A_2 to the imaginary part of three quark scattering.

For the pion, the analogue of Fig. 8 corresponds to antiquark exchange. The question of precocious scaling and the validity of the

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*Possible wrong signature fixed poles complicate the connection.

Bloom-Gilman hypothesis depends, therefore, on the duality properties of quark-antiquark scattering.

To go beyond this statement of logical possibilities requires additional assumptions and will be taken up in the next section. One should not lose sight of the fact that we have found in this model a new way of interpreting both rapid scaling and the Bloom-Gilman hypothesis. This new insight provides the basis for potentially understanding these experimental results from an understanding of quark dynamics.

III. EXCHANGE DEGENERACY AND THE ANNIHILATION RULE

In this section, we attempt to abstract from our understanding of duality in hadronic reactions the properties expected for deep inelastic lepton scattering. We will find that both precocious scaling and the Bloom-Gilman hypothesis for nucleons are a consequence of a simple rule. We will be led to predictions for $e^+e^- \to hX$ as to which hadrons will show rapid scaling and which will not.

It is not our purpose here to present a complete review of Duality.* We would remind the reader, however, that if duality were simply the oft-quoted hypothesis that "resonances build Regge poles and background builds the pomeron," it is well known to fail. There is the famous

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*By Duality here we mean more than the rapid convergence of finite energy sum rules. We refer to the association of specific phenomena (such as resonances) in one channel with the presence or absence of certain exchanges in the cross channel. For a review of conventional discussions of this subject, see Rosner[8] and references therein.

baryon-antibaryon catastrophe, which manifestly violates this hypothesis or requires exotic resonances.[9] In addition, in pomeron-hadron scattering, it has been suggested[10,11] that resonances contribute to the triple-pomeron coupling. The experimental evidence is that there are no clearly established exotic resonances[8,12] and, in any case, meson resonances in baryon-antibaryon annihilation seem to be dual to some low lying exchange and not to the leading trajectories f^0, ω, ρ, A_2.[13] Also there is growing evidence[11,14] for the hypothesis about resonances contributing to the pomeron in pomeron-hadron scattering. Both of these results are consistent with (and were suggested by) duality diagrams. At the same time, the application of duality diagrams to baryons is treacherous. Planarity seems to conflict with the requirement that baryons be symmetric in its quark constituents.[15] The dual pomeron,[16] which works so beautifully for the scattering of mesons, fails to be crossing symmetric in baryon-baryon scattering. Because this last point is not well known, we elaborate on this.

In baryon-baryon scattering, the pomeron is usually drawn as in Fig. 9a. Since there is no net quark exchange, the pomeron is a singlet and, since of even signature, should contribute equally to baryon-antibaryon scattering. However, the analogous diagram for this case has intermediate states with particles of nonzero triality (Fig. 9b). Probably, this representation of baryons is fundamentally incorrect.

Because of the dubious status of arguments about the pomeron based on duality diagrams, we prefer to search for a simpler guide to the known pattern of exchange degeneracy. About the only rule which seems unambiguous and uncontroverted by data is the Annihilation Rule, suggested by Lipkin [17] even before the invention of duality diagrams. As a first approximation, in those days, hadrons were regarded as made up of primarily valence quarks; hadron interactions were regarded as a sum of two-body interactions among the quark constituents of the different hadrons. Lipkin found that the observed pattern of exchange degeneracy followed from the hypothesis that only when the initial and final state hadrons contain a quark and its own antiquark (so that they can annihilate in the isosinglet channel) did the imaginary part of secondary exchanges not vanish. Thus, for example, in the imaginary part of uu, ud, and even $u\bar{d}$ elastic scattering, the four trajectories, f^0, ω, ρ and A_2 cancel each other.[17] However, in $u\bar{u}$ or $d\bar{d}$ scattering, they all add to constructively give a non-zero contribution to the imaginary part. To summarize, strong exchange degeneracy in the t-channel is correlated with the absence of isosinglet quark-antiquark annihilation in the s-channel. This we will call the Annihilation Rule. In hadron scattering, the rule does correspond to the absence of s-channel resonances in meson-meson and meson-baryon scattering. Both the additivity hypothesis and Annihilation Rule are contained in duality diagrams, which however go beyond these simple rules. Here we will assume only additivity and the

Annihilation Rule.

Suppose we applied the rule to the scattering of three quarks. Obviously, there can be no quark-antiquark annihilation, so there will be exchange degeneracy in the crossed channel. However, unlike hadron-hadron scattering, there can be resonances. Of necessity, these resonances must be dual to the pomeron. Consequently, the Annihilation Rule immediately implies both precocious scaling and the Bloom-Gilman hypothesis for the nucleon for x near one. It would be nice if this form of duality were supported by an explicit model of the pomeron. If we assume, in Fig. 9a, that "quarks which don't scatter, don't matter," then by erasing three quarks we arrive at Fig. 10a. If this diagram has a pomeron in the crossed channel, it can also have resonances in the intermediate state. To our surprise then, the form of duality suggested by the Annihilation Rule receives further support from heuristic arguments with duality diagrams for the pomeron. As a matter of principle, we may ask, what about deep inelastic scattering off pions? The picture analogous to Fig. 8 involves quark-antiquark scattering. According to the Annihilation Rule, we would expect there to be secondaries only if the quark and its antiquark could annihilate to an isosinglet. So we would expect the Bloom-Gilman relation to work for example, for $\pi^{\pm}$. Thus as with the nucleon, we expect meson resonances to contribute to the pomeron and for rapid scaling to hold for $\pi^{\pm}$ (at least for x near to one).

However, the case of the π^0, being composed of $u\bar{u}$ and $d\bar{d}$ quarks, is quite different since we can have isosinglet quark-antiquark annihilation. Consequently, we do not expect precocious scaling to hold for the π^0, not even for x near one. All of this carries over to $e^-e^+ \to \pi X$, which is directly measurable at SPEAR. This will be discussed in more detail elsewhere, but the clear implication of the preceding is that, while the charged pions should manifest precocious scaling for x near one, π^0 production will not but, instead, will have important $1/\sqrt{Q^2}$ corrections to scaling. The nonprecocious contribution comes from the isovector photon coupling with the pion to yield an isosinglet missing mass. Analogous remarks hold for other neutral mesons.

What about the Bloom-Gilman relation for these neutral mesons? To answer this requires going beyond the Annihilation Rule. We must know what the pomeron looks like in quark-antiquark scattering, and this question is undoubtedly related to the baryon-antibaryon problem referred to above. Naively, for quark-antiquark scattering, we would be led to draw Fig. 10b; however, such a diagram will never come from a permissible diagram in the dual perturbation theory for hadrons in the way we obtained Fig. 10a from Fig. 9a. Nevertheless, Fig. 10b is appealing, since it does have resonances dual to the pomeron. In these terms, then, deep inelastic scattering from a neutral meson, h^0, for x near one receives the two types of contributions depicted in Fig. 11. In Fig. 11a, we see the scaling contribution having resonances dual to the pomeron. (This same topology occurs for charged pions as well). In

Fig. 11b, we have one of the additional contributions to this process, in which we see isosinglet annihilation of an $u\bar{u}$ pair dual to a reggeon exchange. (The diagram should not be thought of as disconnected but rather as the two halves connected by gluons.) Taken literally, Fig. 11b suggests that only nonresonant production will contribute to the $1/\sqrt{Q^2}$ approach to scaling. If so, the Bloom-Gilman relation will continue to work for h^0, even in the absence of precocious scaling. We have less confidence in these conclusions then in the preceding ones, based as they are on a dubious model of the pomeron. Experiments should first determine whether in e^+e^- annihilation, precocious scaling holds for neutral mesons as it does for charged. If not, then the isoscalar resonances in the missing mass should be analyzed to see whether they contribute only to the scaling function or to both the scaling and nonscaling pieces.

An extremely important and interesting point is that the preceding discussion applies only to the region of x near one, where the dominant exchanges correspond to the valence quarks. Just how far from $x = 1$ these simple exchanges continue to dominate is a dynamical question corresponding to the validity of the valence quark model. It clearly should become worse and worse as $x \to 0$, where exchanges in the hadron-antihadron channel dominate (recall Fig. 4). As the valence quark approximation breaks down, we would expect contributions from quark-antiquark scattering to appear. For scattering from a nucleon, for example, diagrams such as Fig. 12 become as important as Fig. 8. This would give non-scaling ($\nu^{-\frac{1}{2}}$) contributions. This conclusion seems inescapable: A breakdown of the valence quark approximation will be accompanied by a breakdown, not in scaling, but in the rapid approach to the scaling limit.

There is another conceptual difficulty in relating duality in deep inelastic electroproduction to duality in hadronic reactions. Presumably the additivity and Annihilation rules were formulated for the interactions among the quark constituents of hadrons, whereas the coupling of the virtual photon is undoubtedly to "current"quarks. Thus, when one speaks of valence quarks, he should distinguish the two different cases. An example is that a proton consists of uud quarks which is symmetric under interchange of the quark momenta. However, the ratio of νW_2 for neutrons and protons for x near one is close to $\frac{1}{4}$, suggesting that nearly all the momentum is carried by u quarks. Although we do not clearly understand the correspondence, our statement of the breakdown of the valence quark model as $x \to 0$ does not necessarily require modifications to the spectral wave functions.

Naively, we would expect $e^-e^+ \to pX$ to show precocious scaling just as in electroproduction. However, our experience in hadron physics leads us to exercise caution. In peripheral models,[18] which satisfy the

Mueller-Regge analysis employed here, there are threshold effects which cause the production of heavy particles to show significant energy variations. For example, antiproton production at the ISR, $pp \to \bar{p}X$, shows a rapid rise from $s = 450\ \mathrm{GeV}^2$ to $s = 3600\ \mathrm{GeV}.^2$ There is no reason to expect these thresholds effects not to show up also in $e^-e^+ \to pX$, hence, this reaction may not show precocious scaling for an entirely different reason than for light particles such as pions. To put the argument even more simply, deep inelastic muon scattering on iron may manifest precocious scaling over the SLAC energy range, but we doubt that rapid scaling will be observed in $e^-e^+ \to F_eX$.

Further research is required to determine whether the threshold effect is simply kinematical, giving a sort of step-function, or whether it is dynamical and persistant, as in $pp \to \bar{p}X$.

Another interesting process to consider is $e^-e^+ \to \eta X$. As with the π^0, we do not expect this to manifest precocious scaling, owing to the isoscalar contribution to the missing mass. The reaction $e^-e^+ \to KX$ should scale rapidly, for both charged and neutral kaons. Other processes capable of experimental study include $e^-e^+ \to \omega X$ and $e^-e^+ \to \phi X$. Neither of these should scale precociously; however, because these mesons are massive, it may be difficult to ascertain whether it is a threshold effect or not. A word of caution might also surround our prediction that kaons scale rapidly since it takes a heavy cluster to produce a $K\bar{K}$ pair.

Note that, in contradiction to the Harari-Freund picture of hadron scattering, there is no known reason here for the $1/\sqrt{\nu}$ corrections to scaling to be positive. Consequently, scaling cross sections may be approached from below rather than falling to their asymptotic value.

While the data is only preliminary, all the preceding remarks appear to be in accord with the initial reports from SPEAR.[19] In particular, in the production of charged hadrons, $e^- e^+ \to h_c X$, rapid scaling appears to hold for $\frac{1}{2} \lesssim x \leq 1$ but not for the region $0 \leq x \lesssim \frac{1}{2}$ (here $x = \frac{2\nu}{Q^2}$).

Deep inelastic neutrino scattering should be very interesting in this picture. For example, in $\nu \pi^0 \to \mu^- X$, rapid scaling should work (for x near one). However, in $\nu\pi^- \to \mu^- X$, (or $\bar{\nu}\, \pi^+ \to \mu^+ X$) rapid scaling should not hold because of the presence of quark-antiquark annihilation (see Fig. 13). Thus the π^0 and π^- exchange roles from electroproduction. While this is not directly testable, there should be interesting analogous predictions for $\nu N \to \mu^- hX$ and $\bar{\nu}\, N \to \mu^+ hX$ (where h is some particular hadron).* This will be investigated and reported elsewhere.

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*Indeed, as remarked earlier, when h is a nucleon or Δ, one may be able to extract the contribution of the pion pole.

IV. $e^+e^- \rightarrow$ HADRONS

A. Conventional Model

We will assume that electron-positron annihilation proceeds via the single photon channel and, as in electroproduction, the interaction of the photon with hadrons is mediated by a pointlike coupling to quark constituents. (Fig. 14a). As before, we assume quarks are confined so that, the only contribution to the discontinuity of the quark-antiquark scattering amplitude (Fig. 14b) comes from hadrons. Assuming the $q\bar{q}$ amplitude is strongly damped in its transverse momentum, the asymptotic behavior will be given by a Regge limit (Fig. 15), to which the dominant contributions will be the pomeron plus reggeons f^0, ω, ρ, A_2, K^*, K^{**}, ϕ, f'. [See Appendix A for details. We assume that diquark exchange has intercept below zero.] Just as before, since we are dealing with the scattering of a quark with antiquark, by the Annihilation Rule, we do not expect exchange degeneracy to hold. The ratio, R, of the total e^+e^- annihilation cross section, σ^{+}_{tot}, to the cross section for muon pair production will, therefore, have the asymptotic behavior (up to logarithmic corrections)

$$R = R_{\mathbb{P}} + R_R (Q^2)^{-1/2} \quad . \tag{9}$$

The asymptotic constant, in a model with a factorizable pomeron pole, can be written as

$$R_{\mathbb{P}} = \Sigma e_i^2 \beta_i^2 \qquad (10)$$

where the sum extends over different types of quarks. If the pomeron is singlet making equal couplings to all types, we find

$$R_{\mathbb{P}} = \beta^2 R_{free}; \quad R_{free} = \Sigma e_i^2 \quad . \qquad (11)$$

R_{free}, the sum of the squares of quark charges, is the value given by the free quark model. In this model, there seems to be no reason to fix $R_{\mathbb{P}}$ to be equal to R_{free}.*

Let us discuss the approach to scaling. In hadron physics, the Harari-Freund hypothesis implies that corrections to the pomeron contribution to total cross sections should be positive. There seems to be no such analogous statement possible in the case of quark-antiquark scattering. Accordingly, the sign of R_R remains undetermined. Even if the contribution of the Reggeons to forward quark-antiquark scattering could somehow be shown to be positive, there still would be no guarantee that, after integrating over all scattering angles as required for σ_{tot}^{-+}, the resultant contribution would still be positive.

However, there is good reason to expect R_R to be small, which unfortunately would not appear to be in accord with the experimental results.[19] The reason comes from considering the SU_3 generalization

*Similarly there seems no reason for current algebra and parton model sum rules[20] to be satisfied in our model. However, it is logically possible that they are (in agreement with experiment) whereas $R_{\mathbb{P}}$ is $\neq R_{free}$ (which equality is not so hot experimentally).

of the Annihilation Rule, which is that exchange degeneracy holds unless a quark-antiquark pair annihilate not just to an isosinglet but to an SU_3 singlet. Conventionally, the photon is purely in an octet, so the $q\bar{q}$ pairs produced are in an octet state rather than a singlet. Thus in the SU_3 limit, the f^0, ρ, ω, A_2 plus the ϕ, f' contributions are precisely cancelled by the K^*, K^{**} contribution. Hence, $R_R = 0$ in the SU_3 limit. SU_3 symmetry is generally reliable at the level of 20%, so we would be hard pressed to believe R_R can get large enough to account for the data. The only other way out would be for the photon to acquire an SU_3 singlet component, as it does in models with charmed quarks.

B. Some Speculations on σ_{tot}^{-+}

Within the framework of this model, other logical possibilities are possible. Given that the observed ratio R rises so rapidly,[19] perhaps we should entertain modifications of the discussion of Appendix A. Let us inquire about the coupling of Reggeons to hadrons and to quarks. In models for Regge behavior, such as the multiperipheral parton model, the strong damping of the coupling of a Reggeon to a hadron is a direct reflection of the transverse momentum cutoff of the parton distributions in the hadron's wave function in the infinite momentum frame. It is conceivable that a Reggeon's coupling to the quarks themselves is more nearly pointlike. Because one must integrate the quark-antiquark amplitude over all directions to obtain its contribution to the e^-e^+ annihilation cross section, it is possible that σ_{tot}^{-+} would be enhanced. For example, an elementary particle (vector gluon) exchange would yield a multiplicative factor of $\ell n\, Q^2$, whereas pointlike interaction would give an enhancement by an entire power of Q^2. The problem in entertaining such hypotheses is to show that such a dynamical picture would be consistent with the transverse momentum cut off of the hadron's wave function and the experimental observation that the production of hadrons rapidly decreases as a function of momentum. The predictions for multiplicities must also be investigated.

The intriguing thing about this suggestion is that, although a more pointlike coupling will lead to enhancement in σ_{tot}^{-+}, <u>it does not alter the predictions for $e^-h \to e^-X$ or $e^-e^+ \to hX$.</u> In these cases, the transverse momentum damping assumed for the hadron fragmentation into quarks is sufficient to give scaling <u>with no further damping required</u> from the pomeron or Reggeon couplings to quarks.

V. SUMMARY AND SUGGESTIONS FOR FURTHER INVESTIGATIONS

In this paper, we have discussed a parton model for the asymptotic behaviors of weak and electromagnetic interactions. We have imagined that the strong interactions among the quark constituents are quite similar to the strong interactions among hadrons, except that quarks are confined and will never be produced. The virtual constituents always act as if they were very light particles. With a certain additional assumption of damping in the quark's transverse momenta (see assumption

(2) of Appendix A), we found that the Mueller-Regge analysis familiar from hadronic interactions may be applied to these weak and electromagnetic asymptotic limits. From the kinematical standpoint, our model is essentially identical to the Massive Quark Model.[3] All the results of Feynman's formulation of the parton model[2] can be easily obtained here, without having to assume that the quarks are free, except in their electromagnetic or weak couplings. It is easy to see the relation to that discussion; for example, the probability $u(x, p_{\perp})$ of finding an up quark in a proton corresponds here to the limiting fragmentation of a proton into an up quark. Unlike that model based on the handbag diagram (Fig. 17a, see below) we can sensibly discuss hadronic final states. Another distinction concerns the role of the valence quark model. Here, the valence quark model emerges for x near one from the dominance of certain exchanges in quark or diquark channels. How far from $x = 1$ they continue to dominate remains a detailed dynamical question.

Exploiting the fact that all quarks interact, we discussed the approach to the scaling limit, arguing that precocious scaling should be interpreted as exchange degeneracy among the secondary Reggeons. We also pointed out that the Bloom-Gilman relation required that resonances contribute to the pomeron in three quark scattering. Abstracting the Annihilation Rule from discussions of exchange degeneracy in hadron reactions, we demonstrated that precocious scaling is indeed to be expected (for x near one) for deep inelastic electroproduction off a nucleon. What's more, we showed that Bloom-Gilman duality is compatible with the dual model[16] for the pomeron. We went on to argue that precocious scaling should hold for charged pions and all kaons, for x near one, in $e^-e^+ \to \pi^{\pm}X$ or $e^-e^+ \to KX$.

On the basis of the Annihilation Rule, we argued that precocious scaling may <u>not</u> be valid for neutral mesons, not even for x near one. The absence of rapid scaling for $e^-e^+ \to \pi^0 X$ is a crucial test of our model. Similarly, other channels having an isosinglet missing mass may not manifest precocious scaling. These include $e^-e^+ \to \eta X$, $e^-e^+ \to \omega X$, $e^-e^+ \to \phi X$.

In addition, we argued that, for all hadrons, departures from precocious scaling are to be expected for x no longer near one, and the magnitude of the departure can be correlated with the breakdown in the pure valence quark model. Indications from neutrino experiments indicate this should be the case for the nucleon for $x \lesssim 0.3$, as antiquarks begin to compete with valence quarks. The preliminary data from SPEAR[19] show that, for charged hadron production, precocious scaling holds for $0.5 \lesssim x \leq 1$ but not for $0.1 \lesssim x \leq 0.5$, in qualitative agreement with our conclusions.

We considered the total cross section, σ_{tot}^{-+}, for electron-positron annihilation into hadrons, arguing from the Annihilation Rule that this cross section will not manifest precocious scaling. However, neither the magnitude nor sign of the non-scaling terms could be determined.

The ratio R of σ_{tot}^{-+} to $e^-e^+ \to \mu^-\mu^+$ behaves as (up to logarithmic corrections)

$$R \xrightarrow{Q^2 \to \infty} R_P + R_R/Q \quad .$$

Unlike the free quark model, the magnitude of the asymptotic constant is undetermined here. This is interesting in itself, since it demonstrates that the short distance behavior of this model is different from the free quark model. On the other hand, if the handbag diagram (Fig. 17a) were truly cancelled, the equal time structure of this model may violate current algebra. The fixed pole structure[20] of this model is a subject for further investigation. The SU_3 generation of the Annihilation Rule predicts that $R_R = 0$. It would require substantial breaking of SU_3 invariance to account for the rapid energy variation seen in the SPEAR experiment.[19] This seems to us to be an unlikely explanation of the data, so we entertained possible modifications of the model.

Consequently, we reconsidered the dynamical assumptions, indicating a mechanism by which σ_{tot}^{-+} could be enhanced <u>without</u> destroying scaling in $e^-e^+ \to hX$ or deep inelastic scattering $e^-h \to e^-X$. This, too, is an interesting subject for future work.

The application of the model to the asymptotic behavior of the form factor is only one example of the model's applicability to exclusive reactions. It should prove interesting to apply the model to other reactions, such as the large Q^2 behavior of $\mu^-p \to \mu^-N\pi$. A variety of kinematical regimes suggest themselves and comparison with experiment should lead to deeper insight into the model. Other inclusive reactions, such as $e^- N \to e^- hX$, $e^- e^+ \to h_1 h_2 X$, etc., could be analyzed, especially with a view toward their duality properties.

Another fruitful area of application of our duality ideas should be to weak interactions such as $\nu N \to \mu^- hX$. The analysis of weak form factors in reactions such as $\bar{\nu} p \to \mu^+ n$ and $\bar{\nu} p \to \mu^+ p \pi^-$ should be straightforward.

Let us compare our presentation with other models. As has been repeatedly pointed out, our general framework closely resembles the MQM,[3] although we have emphasized dynamical confinement rather than infinitely massive quarks. In the approach of Ref. 3, quark-antiquark scattering manifests pomeron exchange but quark-quark scattering is purely real (for all positive energies). This is a peculiar, and since the pomeron and other Reggeons are not crossing symmetric, one can question the use of Regge theory. Moreover, we believe this leads to self-inconsistencies in the MQM. For example, using only valence quarks, how can the proton-proton total cross section manifest diffraction scattering? Another contradiction appears in deep inelastic lepton scattering from nucleons, since, for x near one, the dominant contribution comes from quark-diquark interactions. If this is not dominated by the same exchanges contributing to quark-antiquark scattering, then νW_2 will not scale. In contrast to this, in our view the quarks are probably effectively light but confined, and there is not necessarily a conflict with crossing symmetry and the Pomeranchuk theorem.

In the MQM,[3] "the quark propagator is a constant, and does not have any pole." For spin one-half quarks, however, the propagator appears to be neither constant nor ($\not{p} + M_q$) but rather $\not{p}$. This seems odd for a theory in which $M_q \to \infty$.

A less fundamental difference from the MQM concerns the valence quark picture. For us, the valence quark model applies only for x near one and finds its justification in the dominance of certain exchanges over others in quark and diquark channels. In contrast thereto, "saturation in terms of the minimum number of quark legs" is regarded as "the most crucial idea" of the MQM.[3]

Our formulation of duality is quite different from that given in the MQM and is our most definitive contribution to the subject. The integration of the ideas familiar from hadronic reactions to weak and electromagnetic phenomena is a beautiful connection made possible by our work.

Our work is quite similar in spirit to that of Kislinger;[21] however, the two-step hypothesis employed there is unnecessary since the Mueller-Regge analysis may be applied directly to the quark-antiquark hadron scattering amplitude.

Although many of our results are the same and diagrams similar, our model differs fundamentally from the constituent interchange model (CIM). One sees this most clearly perhaps in the discussion of form factors, where the asymptotic behavior in our approach arises from a completely different part of phase space from the CIM. Also, we seem to be led to different expectations for the contributions from nonvalence quarks for x near one.

Perhaps the next task is to modify the discussion for spin one-half quarks. Because all particles interact, the generalization is not so straightforward as it might seem. The inclusion of spin will not alter the validity of the theoretical framework or the duality results. As pointed out in Appendix B, it may, however, alter the simple rules for the association of quark exchanges with elementary quarks.

Another area of investigation concerns the extension of these ideas to high energy hadronic interactions involving large momentum transfers, such as the fixed angle limit of single particle inclusive production. It is not clear that the model described here should apply to these phenomena.* A beginning in this description has been taken by Preparata[3] using the MQM assuming valence quarks only.

The most glaring difficulty in confronting experiment concerns

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*However, the application to pp $\rightarrow$ lepton pairs is clear and very similar to normal parton model.[22] There is a neat factorization-like relation between (pp $\rightarrow$ lepton pairs) $\times$ ($e^+e^- \rightarrow X$) and (ep $\rightarrow$ eX) squared.

the infamous data for $e^-e^+ \rightarrow$ hadrons.[19] We have pointed out that, without further assumptions, the model makes no prediction for this reaction. The simple-minded assumption of a transverse momentum cutoff for the quark-antiquark scattering amplitude, when combined with the Annihilation Rule for SU_3, would seem to be in conflict with the observed energy dependence. On the other hand, we point out that a hard component to this scattering amplitude would enhance the prediction for this process, without obviously contradicting the predictions for other reactions.

Regardless of whether the present formulation given in Appendix A survives, we feel the model has already proved its utility in suggesting a number of logical alternatives for the interpretation of precocious scaling and the Bloom-Gilman relation. It provides a general framework for the discussion of a wide class of phenomena and suggests new ways to analyze experiments. At present, its predictions are in qualitative agreement with all known data for deep inelastic scattering and for e^-e^+ annihilation, except for the total cross section for production of hadrons.

VI. RAPIDITY AND THE $Q^2 \rightarrow \infty$ AT FIXED $Q^2/\sqrt{\nu}$ LIMIT

According to our model, νW_2 should be just like the invariant cross section for a hadronic inclusive reaction $qp \rightarrow qX$ integrated over p_T. Now for x near 0, it is usually convenient to discuss the hadronic case in terms of the rapidity variable.

$$y = \frac{1}{2} \log\left(\frac{E + p_{\parallel}}{E - p_{\parallel}}\right).$$

Taking the laboratory frame, with the proton at rest and incident quark along z axis with "lab" momentum ν (remember $s \approx 2m\nu$ is total cm energy squared of incident qp system), we can analogously can define a rapidity variable for inclusive electroproduction by

$$y_{q \text{ final, lab}} = \log\,(m_{Tq}/mx)$$

where m_{Tq} is transverse mass $(\sqrt{m_q^2 + p_T^2})$ of final quark. νW_2 is refreshingly plotted against this new variable in Fig. 16. We have taken the reasonable value $m_{Tq} = 0.5$ GeV. The picture shows the following three regions: proton fragmenting into quark: this is normal fixed x limit and scaling predicts νW_2 to be ν independent in this region. Secondly, we have the pionization limit (Fig. 4) around $y_{cms} = 0$. Note that $y_{cms} = 0$ corresponds to

$$y_{lab} = \frac{1}{2} \log\left(\frac{2\nu}{m_q}\right),$$

where m_q is mass of incident quark (also taken as 0.5 GeV). As described in Appendix A, and indeed is familiar from hadron phenomenology, we also expect scaling in fixed y_{cms} limit. This corresponds to $x \sim \text{const}/\nu^{\frac{1}{2}}$ or $Q^2/\nu^{\frac{1}{2}}$ fixed as ν or $Q^2 \to \infty$. This new limit deserves further investigation when data from Fermilab at high ν becomes available. We can also see (but it may be an optical hallucination as Q^2 is small) the third region of quark to quark fragmentation. We postpone a detailed numerical investigation of the approach to scaling--can we find $\nu^{-\frac{1}{2}}$ behavior at fixed

y_{lab}, $\nu^{-\frac{1}{4}}$ at fixed y_{cms} characteristic of theory--to a later paper. Rather we just remark that the kinematic analogy and hence utility of rapidity variable may be more generally valid than our model with all its shortcomings.

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APPENDIX A

Theoretical Framework

1. General Discussion

To avoid possible confusion, we will make explicit all of our assumptions. Our formulation is quite similar to the MQM,[3] although expressed somewhat more generally. We shall present the model as if there were an underlying field theory, but, as in every other approach to Bjorken scaling, it is unclear whether all of our assumptions are fully realizable in field theory. We suppose that the field theory is expressed in terms of quark fields, so the Hilbert space admits quark and diquark sectors in addition to a physical subspace of hadrons, leptons, and photons.* We imagine that the use of quarks, in addition to the symmetry of the hadron spectrum, is in giving a simple representation for the strong Hamiltonian and for the currents of weak and electromagnetic interactions. Although real quarks may exist, we will take the point of view here that they are a mathematical construction leading to simplifications of the kind referred to but that the quark and diquark sectors are not realized as physical states. Thus,

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*A prototype of the kind of field theory we have in mind is that of colored quarks interacting with colored gluons described in Ref. 24. As there we are agnostic about whether they realize our model, it will be necessary to modify the high energy behavior of the theory and, perhaps, to entertain non-local interactions.

we imagine that our theory is one of confinement[25] with the consequence that the physical subspace is (already) unitary. Precisely how this comes about, we do not know, although a couple of alternatives will be discussed.

To be more concrete, consider the process of deep inelastic electron or muon scattering. We suppose that, for large momentum transfers, the dominant contribution to the coupling of the virtual photon with the quarks is pointlike. In such a field theory, the dominant Feynman diagram for virtual Compton scattering may be subdivided into the two classes indicated in Fig. 17. In 17(a),(the handbag diagram), we group all diagrams in which only one quark interacts with the hadron. In 17(b), we group all diagrams in which both quarks scatter. The physical process for $\ell^- h \to \ell^- X$ is related to the imaginary part of the corresponding amplitude. Clearly the discontinuity of Fig. 17a makes no sense physically, since the discontinuity of the quark propagator and quark-hadron scattering amplitude cannot give physical final states. Since the initial state (photon-hadron or electron-hadron) is physical and we supposed the S-matrix for the physical subspace is unitary, the discontinuity from Fig. 17a cannot contribute to the process in the Laboratory.

Similarly there will be apparently unphysical contributions to the discontinuity of Fig. 17b. In a theory with quark confinement, these make no sense. There would appear to be three possibilities. (1) The model described here is not realizable in conventional field theory.

(2) The discontinuity of Fig. 17a and all unphysical contributions to Fig. 17b must vanish. (3) The discontinuity of Fig. 17a is precisely cancelled by contributions to the discontinuity of Fig. 17b and all other unphysical final states cancel among themselves.

The first alternative seems quite possible, perhaps likely, but then the concept of a Feynman diagram must be defined. We would be at a loss to begin. The second alternative is possible if the threshold for the discontinuity of quark (and diquark) propagators is at infinity so that, in some sense, these propagators are purely real. This is the point of view proposed by Preparata and developed by Gatto and Preparata[3] in the MQM.* Here, energy conservation is invoked to eliminate unphysical final states such as those coming from Fig. 17a. (We call this the kinematical alternative.) The third possibility, which we will refer to as dynamical, would seem characteristic of theories with quark confinement. The "mass" of the quark is not so significant as the fact that it is bound and that the triality zero sectors are purely physical. In this view, a quark and antiquark never exist separately but always attract each other and bind to form a meson.[25] If this is possible, then only hadrons contribute to the discontinuity of the Compton amplitude

*There is the possibility that the threshold is large but finite so that, sufficiently high energy, quarks would be seen in the laboratory. Preparata suggests that the limit in which the quark mass tends to infinity might also be entertained. We think it unlikely that this alternative can be made consistent for reasons given in Sec. V.

(to lowest order in α). The assumption is phenomenological motivated--only leptons, photons and hadrons have been seen in the Laboratory![19]

From this point of view, it does not matter whether Fig. 17a scales or not. Consider Fig. 17b. The imaginary part (which is what is measured) contains hadronic states. According to the reasoning outlined above, we imagine that, in fact, we need only consider the contribution of hadrons to the imaginary part (i.e., quarks are confined). We want to discuss the Bjorken limit of this diagram in detail. For simplicity, we will discuss the case of scalar partons and a scalar hadron (or the amplitude averaged over the hadron's spin states). The kinematical situation is indicated in Fig. 2, for which the corresponding amplitude is

$$T^{\mu\nu} = e_1 e_2 \int d^4k_1\, d^4k_2\, (2k_1 - q)^{\mu} (2k_2 - q)^{\nu}\, T_6^c(k_1, k_1 - q, k_2, k_2 - q, p) \tag{A. 1}$$

e_1 and e_2 are the charges of the quarks k_1 and k_2. In general, there will be a sum over different types of quarks. T_6^c is the connected six-point amplitude, a function of the scalar invariants

$$\begin{gathered} s_j = (p + k_j)^2, \quad t_j = (p + q - k_j)^2, \quad u_j = q^2 \\ m_j^2 = k_j^2, \quad \mu_j^2 = (k_j - q)^2, \quad j = 1,\ 2 \end{gathered} \tag{A. 2}$$

and

$$t_{12} = (k_1 - k_2)^2, \quad u_{12} = (k_1 + k_2 - q)^2 \quad .$$

The missing mass $\mathscr{M}$ is given by

$$\mathscr{M}^2 = (p+q)^2 \; . \tag{A.3}$$

Relations between the invariants above include

$$\begin{aligned} s_j + t_j + u_j &= \mathscr{M}^2 + m^2 + m_j^2 + \mu_j^2 \\ q^2 + t_{12} + u_{12} &= m_1^2 + m_2^2 + \mu_1^2 + \mu_2^2 \; . \end{aligned} \tag{A.4}$$

Consider the contribution to the final state (missing mass) coming from a particular state N and let $A_N(k_j, k_j - q, p, N)$ be the amplitude for this process. For example, for $k_j^0 > 0$, we may interpret A_N as the amplitude for a quark of momentum k_j (and mass m_j) to collide with a proton p producing a quark of momentum $k_j - q$ (and mass μ_j) and a hadron state N. (Here N denotes the collection of all other variables necessary to specify the outgoing hadrons.) Similarly, for $k_j^0 < 0$, we may interpret A_N as the amplitude for antiquark of momentum $q - k_j$ to interact with proton p producing antiquark $-k_j$ and hadrons N. The imaginary part of T_6^c is related to A_N by the optical theorem[26]

$$\mathrm{Im}\, T_6^c = e_1 e_2 \sum_N A_N(k_1, k_1 - q, p, N) A_N(k_2, k_2 - q, p, N)^* \, \delta\,(p+q-N). \tag{A.5}$$

This is the generalization of Mueller's optical theorem to nonforward three-body scattering. If $k_1 = k_2$, this would be related to the inclusive cross section for quark + proton → quark + anything or antiquark + proton → antiquark + anything.

Under assumptions to be stated precisely below, we will show that the Bjorken limit is equivalent to the Mueller-Regge limit of

$\mathrm{Im}\, T_6^c$, in which the proton fragments into either a quark or an antiquark.*

Our assumptions are as follows:

(1) Following Landshoff and Polkinghorne,[1] we assume T_6^c is a rapidly decreasing function of the invariants masses m_j^2, μ_j^2 ($j = 1, 2$). As we shall discuss later, we will be forced to assume a certain analyticity property in these invariant masses which is different from properties established order by order in perturbation theory.

(2) In addition to the preceding assumption, we must assume a cutoff in the transverse momentum of the produced quark or antiquark. To be precise, consider $A_N(k_j, k_j-q, p, N)$. We assume that, in any frame in which the incident hadron and parton are collinear, the amplitude is a decreasing function of the transverse momentum of the produced parton. One way to state this in terms of kinematical invariants is that the amplitude is a decreasing function of $\frac{s_j t_j u_j}{\nu^2}$.

We have not specified the precise rate of decrease in the two cases. In each case, we assume it is sufficiently rapid so that the dominant contribution to the Compton amplitude comes from finite values of μ_j^2, m_j^2, and the transverse momentum or $\frac{s_j t_j u_j}{\nu^2}$.

Although the preceding is manifestly covariant, it is useful for the physical interpretation to use a change of variables reminiscent of Feynman's discussion.[2] Consider the hadron and photon in the frame

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*This observation was first made by Preparata (Ref. 3).

given in Eq. (1). For deep inelastic scattering, we have $0 < x < 1$. We parameterize the loop momenta in Eq. (A.1) as

$$k_j = (k_j^0, \underset{\sim}{K}_j, -x_j P) \tag{A.6}$$

We will be interested in the Bjorken limit: $\nu \to \infty$ for fixed x. By assumption (1) above, the dominant contribution to the integral comes from finite m_j^2, μ_j^2; we then find (for large ν)

$$x_j = x + \frac{\mu_j^2 - m_j^2}{2\nu m} \tag{A.7}$$

Then we may change variables from k_j^0, x_j to μ_j^2, m_j^2

$$d^4k_j = P\, dk_j\, dx_j\, d^2K_j = \frac{P}{4E_j \nu m} d\mu_j^2\, dm_j^2\, d^2K_j \tag{A.8}$$

where

$$E_j \approx \left[x^2P^2 + \underset{\sim}{K}_j^2 + \frac{\mu_j^2 + m_j^2}{2}\right]^{\frac{1}{2}} \tag{A.9}$$

By assumption (2) above, the dominant contribution to the integral comes from finite $\frac{s_j t_j u_j}{\nu^2}$ which, it can be shown, implies that $\underset{\sim}{K}_j^2$ is finite. Hence

$$E_j \approx x\,P + \frac{2\underset{\sim}{K}_j^2 + m_j^2 + \mu_j^2}{4xP}\,. \tag{A.10}$$

For each loop integral there are two cases of interest depending upon the sign of $k_j^0 = \pm E_j$. We denote the four distinct possibilities by

$T_6^{\text{sign } k_1^0, \text{sign } k_2^0}$. Performing the usual decomposition[1,2,3] into structure functions W_1 and W_2, we find for the discontinuity of Eq. (1),

$$W_2 = \frac{e_1 e_2}{4\nu^2 m^2} \int \prod_{j=1}^{2} d\mu_j^2 \, dm_j^2 \, d^2K_j \left[W_6^{++} + W_6^{--} - W_6^{+-} - W_6^{-+} \right] \tag{A.11}$$

where W_6 denotes the discontinuity of T_6 in $\mathscr{M}^2$. The kinematical invariants on which W_6 depends become simple in these variables. For example, for W_6^{++}, we find

$$\begin{aligned} s_j^+ &\approx 2\nu m, \qquad u_j = -2x\nu m \\ t_j^+ &= m^2(1-x) - \left(\frac{\underset{\sim}{K}_j^2 + \mu_j^2(1-x)}{x} \right) \\ t_{12}^{++} &= -(\underset{\sim}{K}_1 - \underset{\sim}{K}_2)^2 \end{aligned} \tag{A.12}$$

We now recognize that the Bjorken limit is identical with the Mueller-Regge limit of this discontinuity of the three-to-three scattering. We further recognize this limit as the proton fragmentation limit (see Fig. 3). Thus, the asymptotic behavior will be governed by the leading singularities in the t_{12} channel. For simplicity, we will speak as if these singularities were Regge poles but <u>the basic kinematical results in no way depend on this assumption.</u> The asymptotic behavior then can be written as

$$W_6^{++} \to \beta_{\alpha_a}(m_1^2, m_2^2, t_{12})\, f_{\alpha_a}(\mu_1^2, \mu_2^2; x, \underset{\sim}{K}_1^2, \underset{\sim}{K}_2^2, t_{12})\,(2x\nu)^{\alpha_a(t_{12})}. \tag{A.13}$$

Recalling Eq. (A.11), we see that if the leading singularity in the quark-antiquark channel has intercept one at $t_{12} = 0$, then we find scaling

$$\nu W_2(\nu, q^2) \to \Sigma\, e_i^2\, F_2^i(x) \tag{A.14}$$

up to logarithmic corrections. Note that we obtain exact scaling if, and only if, the leading singularity is a fixed pole at one, for which we have no reason to argue. In this model, logarithmic violations of scaling seem the most natural result.

In the sort of bastardized field theory we have postulated, there is no reason a priori not to associated the leading singularities in the quark-antiquark channel with the same singularities exchanged in hadron channels. It certainly would seem unusual for Reggeons such as the f^0 or ρ not to couple but the pomeron is something else again. We usually think of this as arising in some self-consistant way from the requirements of unitarity of the strong interaction S-matrix. Whether it should appear in quark-quark scattering is unclear. Even if it were a factorizable pole in the angular momentum plane, it would not necessarily couple to quarks. Discussions of the duality properties of hadron amplitudes, however, generally include the tacit assumption that the pomeron couples to quarks. We will assume this here.

If we do so, then the singularity at one is to be identified with the pomeron and the question of the exact or approximate scaling has been related to the precise nature of diffraction scatteing. This connection, if true, would appear to be very far reaching. In another language, we would say that the precise structure of the singularity of current commutators on the light cone in coordinate space is determined by the well-known long range interaction in momentum spa ce associated with diffraction scattering.

Unfortunately, we do not know the precise nature of the pomeron in hadron scattering. Even if we did, the scale over which the asymptotic behavior would be achieved may not be simple to establish. Let us recall the analogous problem for hadron scattering. As a matter of principle, we would expect single particle inclusive cross sections $\frac{d\sigma}{dp_\perp^2 dx}$ to manifest the same asymptotic behavior as total cross sections, yet, at the ISR, they seemed to be less energy dependent then the pp total cross section. In general, we expect the asymptotic behavior of νW_2 to be the same as the asymptotic behavior of hadronic elastic scattering cross sections.

There is a technical point ignored above which we must mention, viz, in perturbation theory, the analyticity of T_6 in the invariant mass (or of $\beta_\alpha(m_1^2, m_1^2, t_{12})$) would be such that the integrals over m_1^2 and m_2^2 could be displaced to infinity without intersecting any singularities. Consequently, this contribution to W_6^{++} would give a zero contribution to W_2. To avoid this, we must assume that the contour deformation cannot be carried out. The most "natural" hypothesis is to assume very strong damping in the invariant masses, e.g., such as

$$e^{-c(m^2)^2}$$

so that there is an essential singularity at infinity. We explicity make such an hypothesis. Perhaps it is not so surprising that the analyticity in a theory with confinement of quarks would be very different from perturbation theory. In any case, if we wish to retain the description of the parton model given by Feynman,[2] physical processes should only depend on finite (and small) masses of the virtual quarks. Unlike previous descriptions, however, $F_2(x)$ is not strictly the probability of finding a quark with longitudinal momentum fraction x, but is related to the cross section for fragmentation of the hadron into the quark. To the extent that the pomeron factorizes, however, the fragmentation probability f_α (See Eq. (A.13)) is a property of the hadron alone.

So far we have discussed only T_6^{++}. The discussion of T_6^{--} is obviously quite similar: $q-k_1$ and $q-k_2$ are incoming antiquarks and $-k_1$ and $-k_2$ are outgoing antiquarks. T_6^{+-} and T_6^{-+} are quite different, however, since the leading singularities in the $(k_1+k_2-q)^2$ channel will control the asymptotic behavior. This is a diquark exchange, whose intercept, though unknown, will be assumed to be less than or equal to zero, so it gives a correction of order $\frac{1}{\nu}$ or less to scaling. (See the discussion below of the proton form factor for further comments on diquark exchange.)

This completes the theoretical framework for deep inelastic lepton scattering. So far, we have discussed only the leading contribution coming from the pomeron. Returning to Eq. (A.13), in addition to the pomeron, we should sum over the next to leading singularities f^0, ρ, ω, A_2. With intercepts near 1/2, these obviously give $\nu^{-1/2}$ corrections to the scaling contribution coming from the pomeron.

2. Behavior as $x \to 0$

As is well known in the discussion of inclusive hadronic reactions, the behavior as $x \to 0$ passes continuously over to the pionization limit. For x wee (of order $1/\sqrt{\nu}$) and fixed K_j^2, we have $u_j \sim \sqrt{\nu}$ and $t_j^+ \sim \sqrt{\nu}$. As t_j becomes large, the fragmentation function f_α (Eq. A.13) is dominated by leading singularities in the hadron-antihadron channel (see Fig. 4).

$$f_{\alpha_a} \to \beta_{pp}^{\alpha_b}(0)\, f_{\alpha_a \alpha_b}(\mu_1^2, \mu_2^2; \underset{\sim}{K}_1^2, \underset{\sim}{K}_2^2, t_{12})(2x)^{-\alpha_b(0)} . \tag{A.15}$$

Consequently

$$W_6^{++} \to \beta_{pp}^{\alpha_b}(0)\, \beta_{\alpha_a}(m_1^2, m_2^2, t_{12}) f_{\alpha_a \alpha_b}(\mu_1^2, \mu_2^2; \underset{\sim}{K}_1^2, \underset{\sim}{K}_2^2, t_{12})(2x\nu)^{\alpha_a(t_{12})}(2x)^{-\alpha_b(0)} . \tag{A.16}$$

If the leading singularities α_a, α_b have intercept one, then, up to logarithms, we obtain the well known result,

$$F_2(x) \to \text{constant as } x \to 0$$

More generally, if we add secondary trajectories $\alpha_b(0)$ with intercept $1/2$ (f^0, ω, ρ, A_2), we obtain Eq. (6) of the text. So far the discussion is quite similar to the familiar discussion of the handbag; however, here as x becomes wee, we explicitly see the relationship to the central plateau of hadron pionization. This point is elaborated in Sec. VI.

3. Behavior as $x \to 1$

As $x \to 1$, the dominant contribution to f_{α_a} will come from the leading exchanges in the hadron-antiquark channels (see Fig. 5)

$$f_{\alpha_a}(\mu_1^2, \mu_2^2; x, \underset{\sim}{K}_1^2, \underset{\sim}{K}_2^2, t_{12}) \xrightarrow{x \to 1} \beta(\mu_1^2, \underset{\sim}{K}_1^2)$$
$$\times\, g(\underset{\sim}{K}_1^2, \underset{\sim}{K}_2^2, t_{12}) \beta(\mu_2^2, \underset{\sim}{K}_2^2)(1-x)^{\alpha_a(t_{12}) - \alpha(-\underset{\sim}{K}_1^2) - \alpha(-\underset{\sim}{K}_2^2)} . \qquad (A.17)$$

Consequently, up to possible logarithmic corrections, we find a contribution to $F_2(x)$ given in Eq.(A. 7) of the text. A similar discussion obviously applies to W_6^{--} and W_6^{-+}.

4. Asymptotic Behavior of Form Factors

Among the final states occurring in Fig. 1b are the hadron resonances. Since these occur for fixed values of the missing mass, $\mathscr{M}^2 = (1-x)\nu m + m^2$, they correspond to the region $x \sim 1$. It is clearly that the model for the form factor may be obtained from Fig. 14b by going to the pole in the missing mass as shown in Fig. 6a. The form factor will be given by

$$(p+p')^\mu F(q^2) = e_1 \int d^4k_1 (2k_1 - q)^\mu T_4^c(k_1, k_1 - q, p, p') . \qquad (A.18)$$

Let us assume as before, that T_4 is such that the dominant contribution as $q^2 \to \infty$ comes from finite values of $m_1^2 = k_1^2$, $\mu_1^2 = (k_1 - q)^2$, and the transverse momentum. We then find that the asymptotic behavior is given by a Regge limit of T_4^c. If the sign of k^0 is positive, it corresponds to backward scattering in quark (k) + hadron (p) → quark (k-q) + hadron (p'). [If $k^0 < 0$, it corresponds to antiquark-hadron elastic scattering in the backward direction.] We then write the asymptotic behavior as

$$T_4^+ \to \beta(m_1^2, \underset{\sim}{K}_1^2)\, \beta(\mu_1^2, \underset{\sim}{K}_1^2)(-q^2)^{\alpha_{1p'}(-\underset{\sim}{K}_1^2)} ,$$

$$T_4^- \to \bar{\beta}(\mu_1^2, \underset{\sim}{K}_1^2)\, \bar{\beta}(m_1^2, \underset{\sim}{K}_1^2)(-q^2)^{\alpha_{1\bar{p}}(-\underset{\sim}{K}_1^2)} . \qquad \text{(A. 19)}$$

For a pion, $\alpha_{1p'}$ corresponds to quark exchange channel; $\alpha_{1\bar{p}}$, to antiquark exchange. For a nucleon, $\alpha_{1p'}$ is a two-quark channel; $\alpha_{1\bar{p}}$, a four-quark channel. Inserting this into Eq. (A. 18), we find for the asymptotic behavior of the form factor

$$F(q^2) \to \frac{e_1}{2}\int d^2K_1 \left[\beta(\underset{\sim}{K}_1^2)^2(-q^2)^{\alpha_{1p'}(-\underset{\sim}{K}_1^2)-1} + \beta(\underset{\sim}{K}_1^2)^2(-q^2)^{\alpha_{1p'}(-\underset{\sim}{K}_1^2)-1}\right], \qquad \text{(A. 20)}$$

where

$$\beta(\underset{\sim}{K}^2) = \int dm^2\, \beta(m^2, \underset{\sim}{K}^2) .$$

(The final form is appropriate only for a factorizable exchange, but its modification for the general case is obvious.)

From Eq. (A.20) we obtain Eq. (8), up to possible logarithmic factors. When compared with Eq. (7), we get the Drell-Yan-West relation very naturally. While such a relation is possible for the handbag diagram, it is not at all necessary. It arises naturally here only because we are dealing with a connected three-body amplitude.

There is an important point which has been suppressed above concerning the signature of the exchange. One would expect there to be another term in the asymptotic behavior of T_4^{++} of the form $(-2p\cdot k_1)^{\alpha_{1p'}}$, owing to the s-channel cut, in addition to the $(-q^2)^{\alpha_{1p'}}$ coming from the u-channel. In ordinary field theory, the s-channel states do not show up as singularities in the form factor, that is, the signularities of the integrand are not singularities in q^2. Consequently, the form factor is real for spacelike q^2. It is certainly in the spirit of our picture of confinement that singularities in channels with non-zero triality do not contribute to singularities in physical amplitudes. We would like only hadronic states in the quark + antiquark → hadron + antihadron channel to contribute to singularities of the form factor. As a technical matter, it is worrisome however, since our assumption concerning the inability to rotate the contours of integration in the quark invariant masses might give new singularities in the form factor. Since the reality of the form factor for spacelike q^2 follows in field theory from microcausality and positivity of the support for the invariant mass-squared $p_\mu p^\mu = P^2$ one would not like to disturb this result. (The same problem arises in the discussion of the photon propagator (polarization tensor) below.) This point requires further investigation, but it seems likely that the model as presently formulated contradicts microcausality and the support of P^2.

5. $e^-e^+ \to$ Hadrons

We have summarized our assumptions for this process in the text. We must compute the imaginary part of the vacuum polarization tensor which, by Fig. 14b, is given by

$$\Pi^{\mu\nu} = e_1 e_2 \int d^4k_1\, d^4k_2 (2k_1 - q)^\mu (2k_2 - q)^\nu\, T_4(k_1, k_1 - q, k_2, k_2 - q)\,, \qquad (A.21)$$

e_1 and e_2 are the charges of the quarks k_1 and k_2, respectively. Generally, we must sum over all types of quarks. The quark-antiquark elastic scattering amplitude, T_4, is generally a function of the kinematical invariants $m_j^2 = k_j^2$, $\mu_j^2 = (k_j - q)^2$ $(j = 1, 2)$ as well as q^2, $t_{12} = (k_1 - k_2)^2$, and $u_{12} = (k_1 + k_2 - q)^2$. (Note that $q^2 + t_{12} + u_{12} = m_1^2 + m_2^2 + \mu_1^2 + \mu_2^2$).

We are interested in the large q^2 behavior of T_4. As before, our first assumption is damping in the invariant masses of the quarks. Secondly, let us assume that T_4 is a rapidly decreasing function of the momentum transfer $\dfrac{t_{12} u_{12}}{q^2}$. Then the asymptotic behavior of T_4 will be given by Regge limit and the dominant contributions to the integral comes from finite ranges in t_{12} and u_{12}. It is convenient to analyze the contributions in the center-of-mass frame where $q = (Q, \vec{0})$. The exchanges in the t_{12} channel includes the pomeron and reggeons f^0, ω, ρ, A_2 (see Fig. 15). In a model having several types of quarks, t_{12} may also correspond to other exchanges, coming from "off-diagonal" terms, such as charged ρ or A_2 and K^* or K^{**}. (The strange exchanges will be lower lying then the nonstrange.) The u_{12} channel corresponds to diquark exchanges, which we have assumed to have intercept below zero. Keeping only the pomeron contribution for now, we find that the imaginary part of $\Pi^{\mu\nu}$ goes as (See Fig. 15)

$$\rho(Q^2) \to e_1 e_2 \int dt_{12}\, \beta(t_{12})^2 (Q^2)^{\alpha_P(t_{12}) - 1} \tag{A.22}$$

where $\mathrm{Im}\Pi^{\mu\nu} = \left(-g^{\mu\nu} q^2 + q^{\mu} q^{\nu}\right)\rho(q^2)$. Only diagonal terms contribute to the pomeron exchange, so $e_1 = e_2$. Consequently, up to possible logarithmic corrections, $\rho(Q^2) \to$ constant as $Q^2 \to \infty$. The total cross section for $e^- e^+ \to$ hadrons is related to ρ by

$$\sigma_{tot} = \frac{8\pi^2\alpha^2}{Q^2}\,\rho(Q^2)\,. \tag{A.23}$$

Consequently, we find for the total cross section, $\sigma_{tot} \to (Q^2)^{-1}$ (up to logarithmic factors.) In another language, we would say that the singularity <u>structure</u> of the short distance expansion of the product cf currents is the same as in the free quark model, but the overall normalization is altered. If the pomeron is not a fixed pole, then even the singularity structure is modified by logarithmic factors.

If we write $\beta(t_{12})^2 = \beta_i^2 b e^{bt_{12}}$, then we obtain Eq. (10).

APPENDIX B

Elementary Quark Exchange

In the preceding, we have found that the asymptotic behavior of the hadron form factors and the behavior as $x \to 1$ of the scaling functions $F_2(x)$ are determined by exchanges α in channels with quark or diquark quantum numbers. The values of these trajectories are not given within the model presented. An attractive hypothesis would be that the leading singularities in these channels come from the exchange of an elementary quark or two elementary quarks, respectively. With scalar quarks, for the pion form factor, elementary quark exchange ($\alpha(t_{12}) = 0$) leads to

$$F_\pi(q^2) \to (q^2)^{-1} \tag{B.1}$$

For a spinless nucleon, the exchange of two quarks gives $\alpha_{1p'} = \alpha + \alpha - 1 = -1$, so that

$$F_N(q^2) \to (q^2)^{-2} \tag{B.2}$$

a dipole falloff. Note that, with elementary exchanges, we obtain integral powers, without any logarithmic factors. It is at least amusing that the results obtained with scalar quarks correspond to the simple power counting rules of Brodsky and Farrar.[27] Consider another example, the contribution to $F_2(x)$ for a nucleon from the fragmentation into an antiquark. For x near one, for a proton to fragment to an antiquark requires the exchange of at least four quarks. (See Fig. 15). Four elementary quarks lead to $\alpha_{\text{eff}} = 4\alpha - 3 = -3$. Therefore, this contribution to $F_2(x)$ would be of the form[22] $(1-x)^{1-2\alpha_{\text{eff}}} = (1-x)^7$. However, there is another way four quarks can be exchanged, that is, if three of them bind to form a Regge pole. For example, a proton could fragment into a $\bar{u}$ antiquark by exchanging an elementary quark and a nucleon or Δ. This leads e.g., to $\alpha_{\text{eff}} = \alpha + \alpha_\Delta - 1 = \alpha_\Delta - 1$. Since $\alpha_\Delta \approx 0.2$, this gives $\alpha_{\text{eff}} = -0.8$, which, in turn, gives a contribution to $F_2(x)$ of the form $(1-x)^{2.6}$. This dominates not only four free quarks, but also the contribution $(1-x)^3$ from valence quarks! Unfortunately we cannot compute the relative strengths of the different exchanges, so we cannot say which will dominate. It is clear, however, that the effect of three quarks forming a bound state (Reggeon) would not appear in an analysis of perturbation theory graphs and, consequently, the conclusions of Brodsky and Farrar[22,27] are suspect.

Unfortunately, the case of spin one-half quarks is complicated and still under investigation. It appears, however, that while we get a monopole form factor for the pion, we also get a monopole for the nucleon since $\alpha_{\text{eff}} = \frac{1}{2} + \frac{1}{2} - 1 = 0$. Consequently, the beautiful results obtained with scalars seem coincidental.

REFERENCES

[1] P. V. Landshoff and J. C. Polkinghorne, Physics Reports 5 (1972) 1: P. V. Landshoff, J. C. Polkinghorne and R. D. Short, Nucl. Phys. B28 (1971) 225.

[2] R. P. Feynman, Photon-Hadron Interactions, W. A. Benjamin, Inc., Reading, Mass., 1972.

[3] G. Preparata, Phys. Rev. D7, 2973 (1973); CERN Preprints, Ref. TH. 1836-CERN, 20 March 1974 and Ref. TH. 1859 CERN, 29 April 1974. See also R. Gatto and G. Preparata, Nucl. Phys. B67, 362 (1973); Preprints: Brookhaven National Laboratory BNL-18228, September 1973; Ref. TH. 1825-CERN, 31 March 1974

[4] S. D. Drell and T. M. Yan, Phys. Rev. Letters 24, (1970) 181; G. West, Phys. Rev. Letters 24 (1970) 1206.

[5] A. Bodek et al., "Observed Deviations from Scaling of the Proton Electromagnetic Structure Functions," SLAC-Pub-1442 (1974).

[6] E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25 (1970) 1140 and Phys. Rev. D4 (1971) 2901.

[7] M. B. Einhorn, J. Ellis, and J. Finkelstein, Phys. Rev. D5 (1972) 2063.

[8] J. L. Rosner, "The Classification and Decay of Resonant Particles," SLAC-Pub-1391 (1974).

[9] J. L. Rosner, Phys. Rev. Lett. 22 (1969) 689.

[10] M. B. Einhorn, M. B. Green and M. A. Virasoro, Phys. Lett. 37B (1971) 292; Phys. Rev. D6 (1972) 1675, Phys. Rev. D7 (1973) 102.

[11] R. D. Field and G. C. Fox, "Triple Regge and Finite Mass Sum Rule Analysis of the Inclusive Reaction pp $\to$ p X", CALT-68-434 (1974).

[12] J. L. Rosner in 1970 Philadelphia Conference, edited by C. Baltay and A. H. Rosenfeld.

[13] P. Hoyer, R. G. Roberts and D. P. Roy, Physics Letters 44B (1973) 258.

[14] C. Quigg, "Meson Spectroscopy and the Phenomenology of High Energy Collisions," IV International Conference on Meson Spectroscopy, Boston (1974).

[15] M. B. Einhorn, "Application of Dual Resonance Models to Inclusive Reactions," Vol. 1, Proceedings of XVI International Conference on High Energy Physics, Sept. 6-13, 1972; (J. D. Jackson and A. Roberts, eds.); National Accelerator Laboratory, Batavia, Illinois.

[16] P. G. O. Freund and R. J. Rivers, Phys. Lett. 29B (1969) 510; P. G. O. Freund, Nuovo Cimento Letters 4 (1970) 147.

[17] H. J. Lipkin, Phys. Rev. Letters 16 (1966) 1015.

[18] M. B. Einhorn and S. Nussinov, Phys. Rev. D, to be published.

[19] LBL-SLAC SPEAR Magnetic Detector Collaboration, data presented at Washington, D.C., meeting of the American Physical Society, April 1974.

[20] In addition to current algebra fixed poles, there is the subject of those at $j = 0$, characteristic of the handbag diagram. See S.J. Brodsky, F.E. Close and J.F. Gunion, Phys. Rev. D8, (1973) 3678.

[21] M. B. Kislinger, Phys. Lett. 46B (1973) 225.

[22] G.R. Farrar, "Parton Distributions in Hadrons and the Cross Sections for Lepton Pair Production," CALT-68-422 (1974).

[23] E.D. Bloom, "Recent Results Concerning the Electromagnetic Structure of the nucleon," SLAC-Pub-1319 (1973).

[24] H. Fritzch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

[25] A. Chodos, et al., Massachusetts Institute of Technology Preprints MIT-CTP-387-Rev., March 1974 and MIT-CTP-417.

[26] A.H. Mueller, Physical Review D2 (1970) 2963; J.C. Polkinghorne, Nuovo Cimento 7A (1972) 555, H.P. Stapp, Phys. Rev. D3 (1971) 3177, C.I. Tan, Phys. Rev. D4, (1971) 2412.

[27] S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. 31 (1973) 1153.

FIGURE CAPTIONS

Fig. 1 Deep Inelastic Lepton Scattering (DILS) from a hadron target, producing hadrons.

Fig. 2 DILS as a discontinuity of three-body scattering.

Fig. 3 Bjorken limit as a Mueller-Regge limit.

Fig. 4 Behavior of fragmentation for $x \to 0$ as a pionization limit.

Fig. 5 Behavior of fragmentation for $x \to 1$ as a triple-Regge limit.

Fig. 6 (a) Contribution of particle or resonance to DILS.

(b) Asymptotic behavior of form factor as a Regge limit.

Fig. 7 The three contributions to fragmentation ($h \equiv$ hadron, $q \equiv$ quark, $\bar{q} \equiv$ antiquark)

a) $(h \xrightarrow{q_1} q'_1)(h \xrightarrow{q_2} q'_2)^*$

b) $(h \xrightarrow{q'_1} \bar{q}_1)(h \xrightarrow{q'_2} \bar{q}_2)^*$

c) $(h \xrightarrow{q_1} q'_1)(h \xrightarrow{q'_2} \bar{q}_2)^*$

Fig. 8 Relation between finite missing mass and scaling limit.

Fig. 9 (a) The dual pomeron for BB scattering.

(b) Analogous diagram for $B\bar{B}$ scattering.

Fig. 10 (a) Dual pomeron in three-quark scattering.

(b) Pomeron (?) in quark-antiquark scattering.

Fig. 11 Speculations about duality for the π^0:

(a) The pomeron (?)

(b) Secondary reggeons dual to gluons.

Fig. 12 Deviations from precocious scaling occur as non-valence quarks appear.

Fig. 13 $\nu\pi^- \to \mu^- X$ may not scale precociously.

Fig. 14 (a) electron-positron annihilation to hadrons.

(b) Relation to discontinuity of quark-antiquark scattering.

Fig. 15 Asymptotic behavior of $e^- e^+ \to x$ as a Regge limit.

Fig. 16 νw_2 for electroproduction plotted at fixed $\nu = 5$ GeV against lab rapidity y defined in Sec. VI. We also mark y_{1ab} value for which cms rapidity is zero. Data is taken from compilation in Ref. 23 and is extracted assuming R = 0.168 from measured cross sections.

Fig. 17 Feynman diagrams in a quark field theory

(a) Handbag diagram used in Ref. 1

(b) Pussycat diagram considered in this paper.

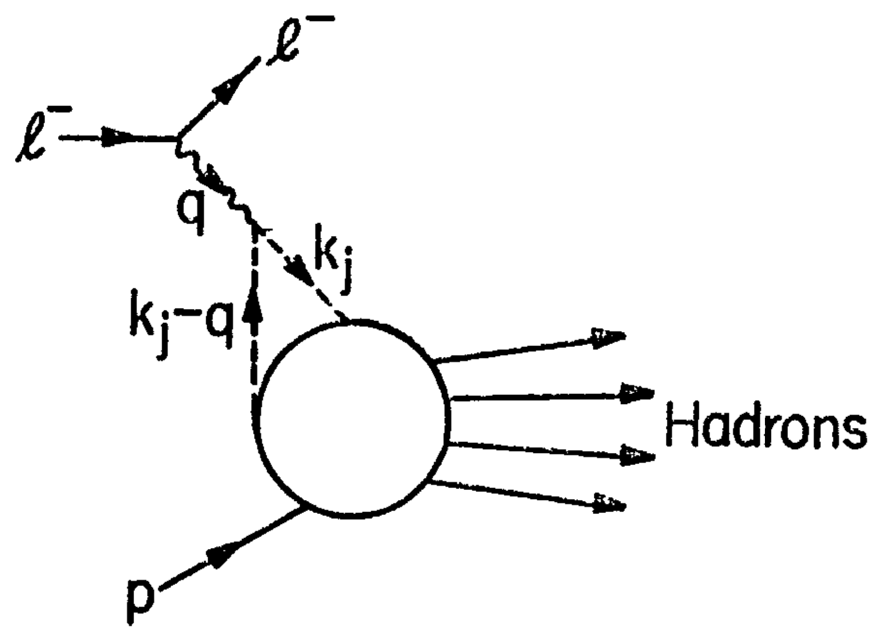
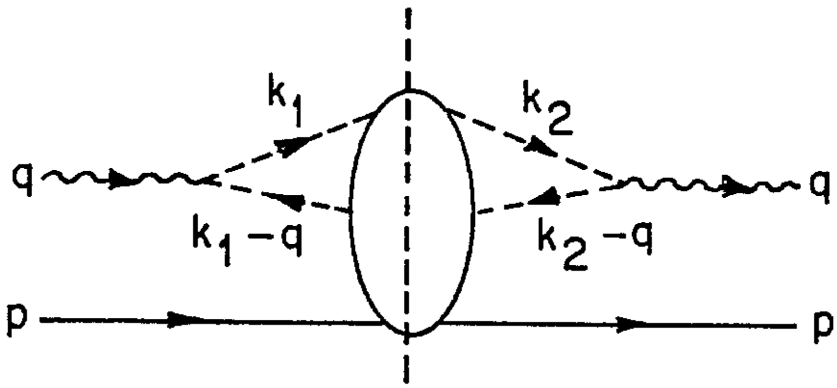


Fig. 1



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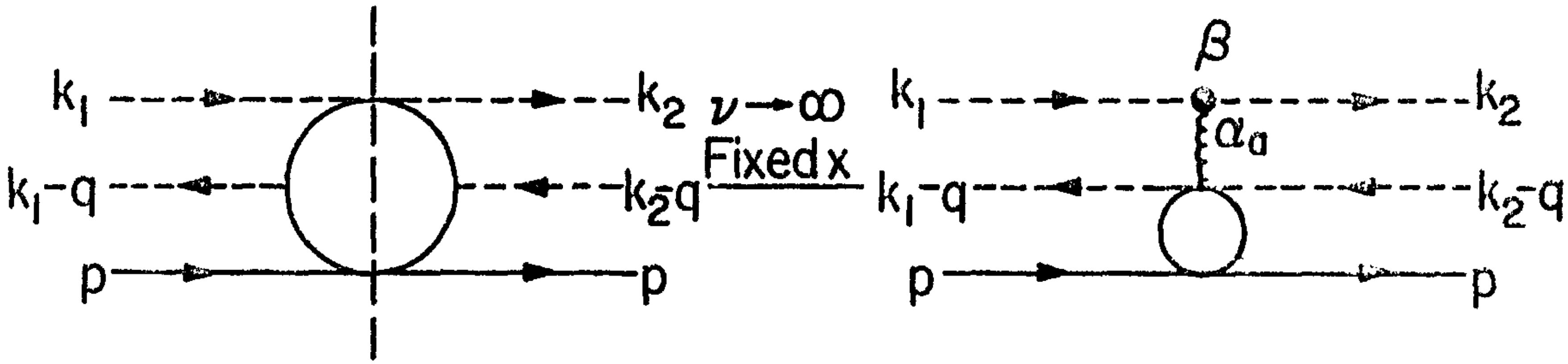

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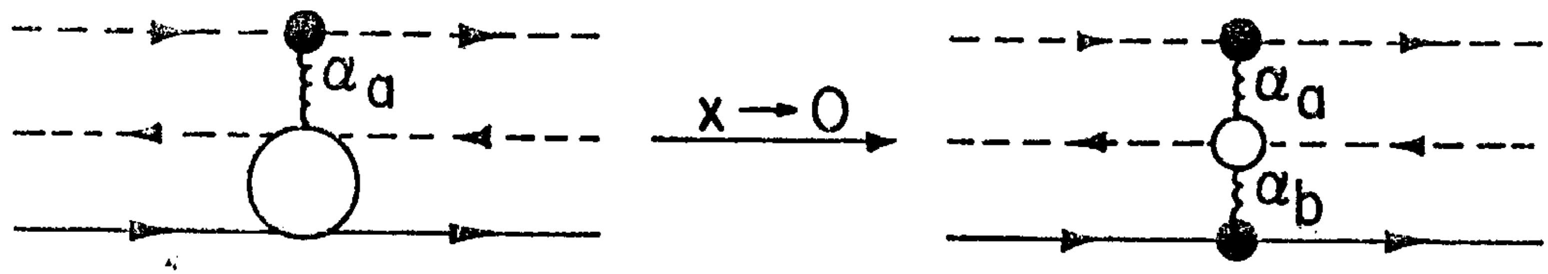

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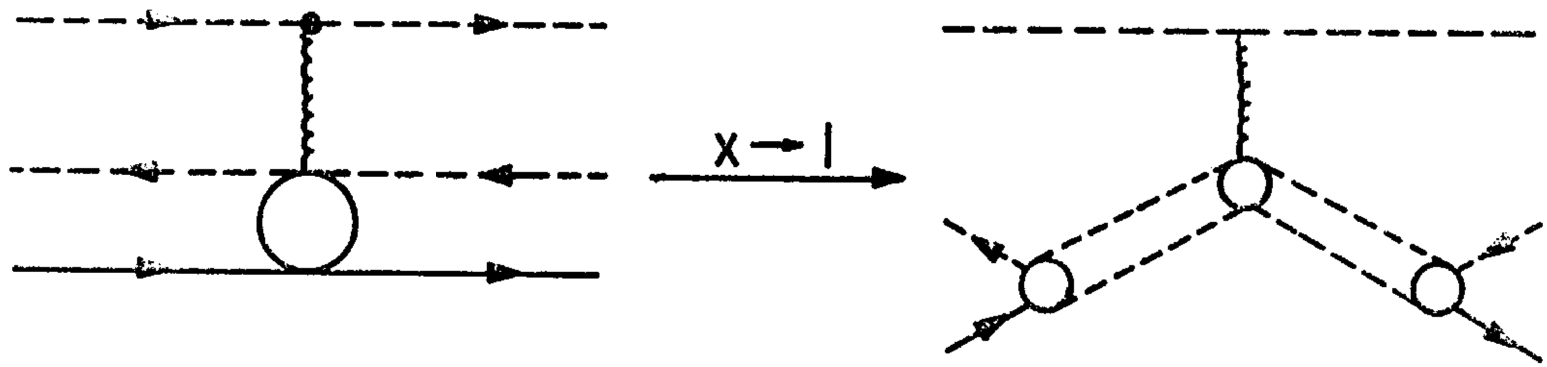


Fig. 5

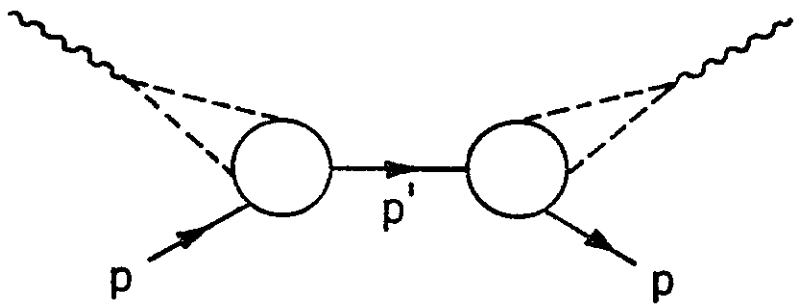


Fig. 6a

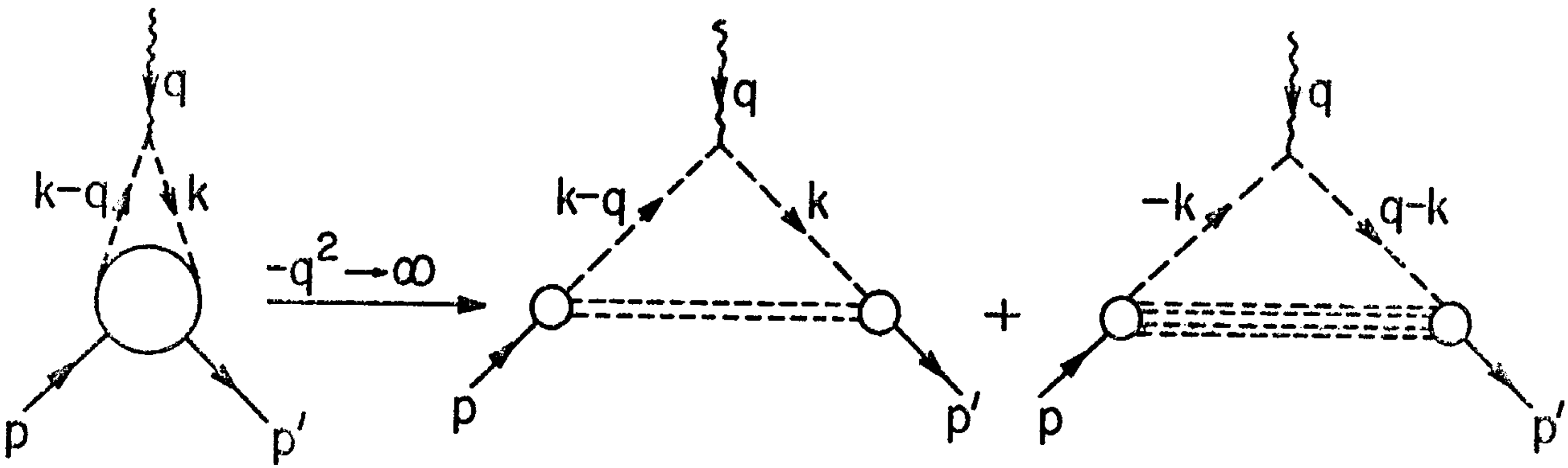


Fig. 6b

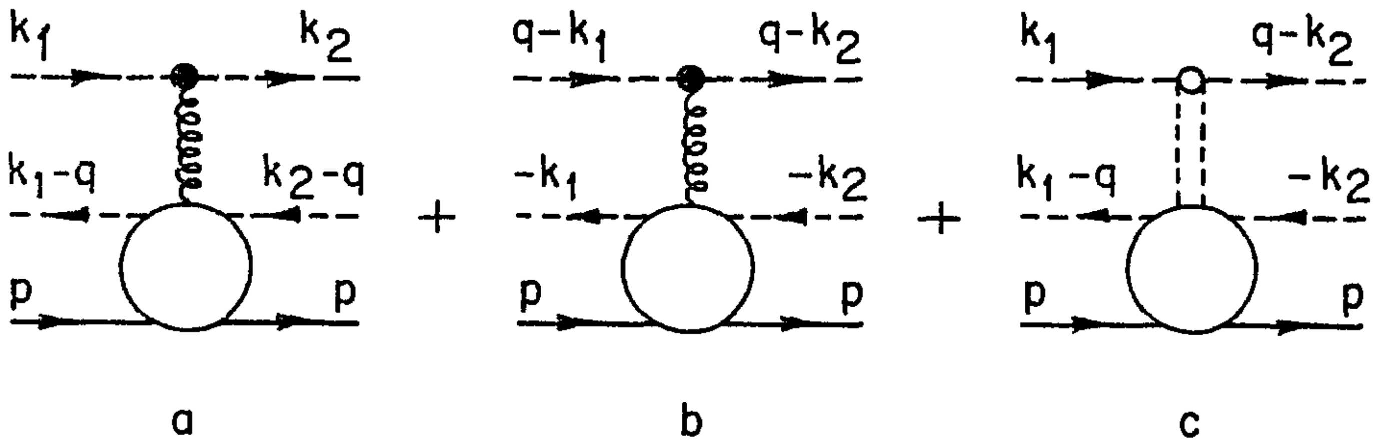


Fig. 7

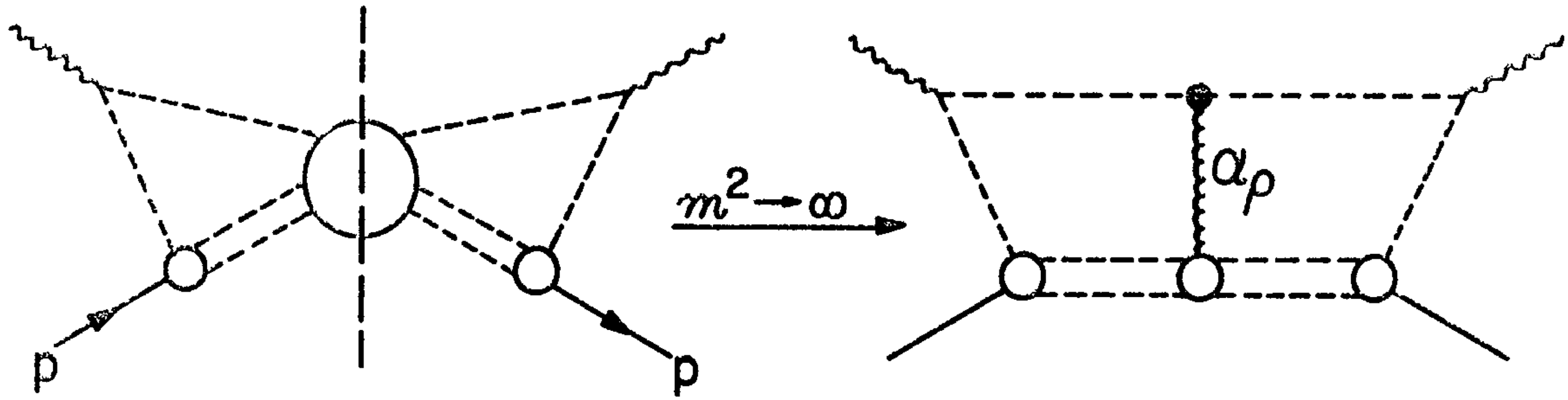


Fig. 8

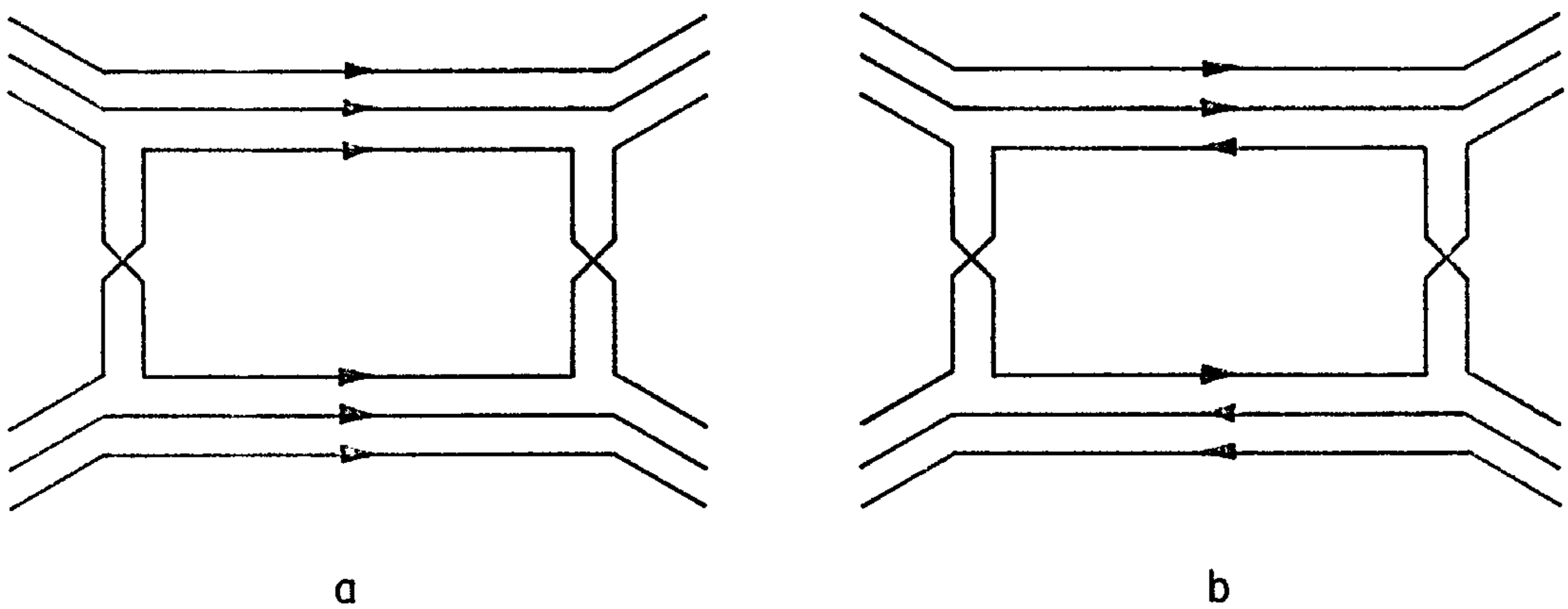


Fig. 9

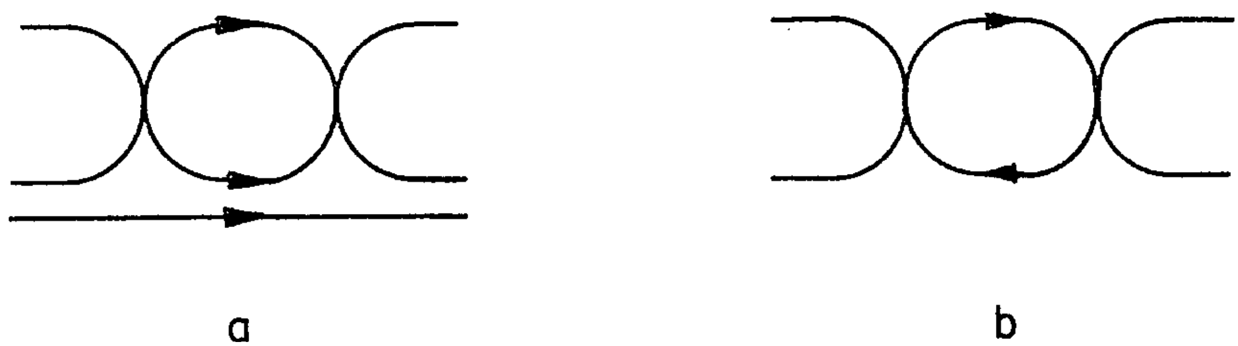


Fig. 10

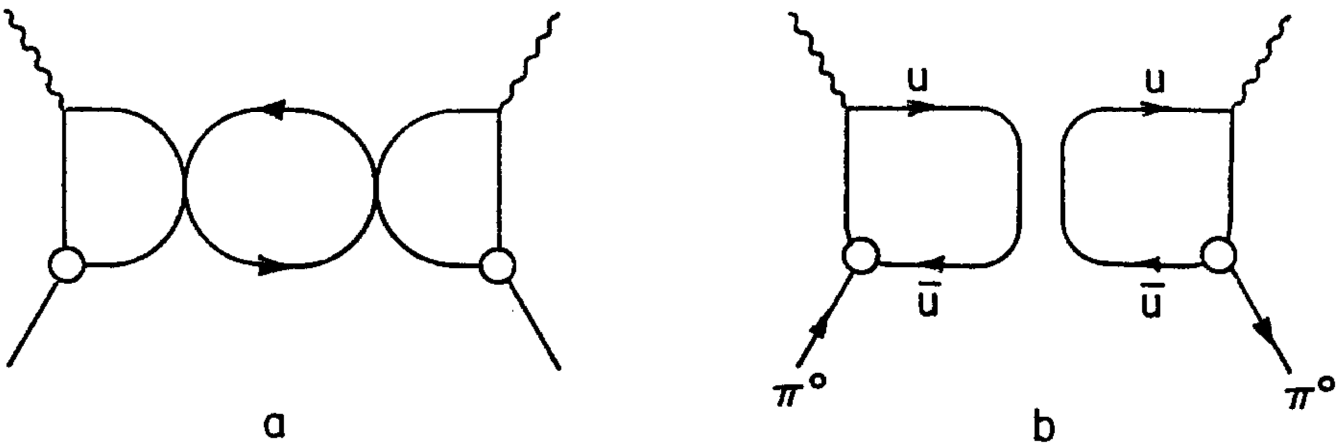


Fig. 11

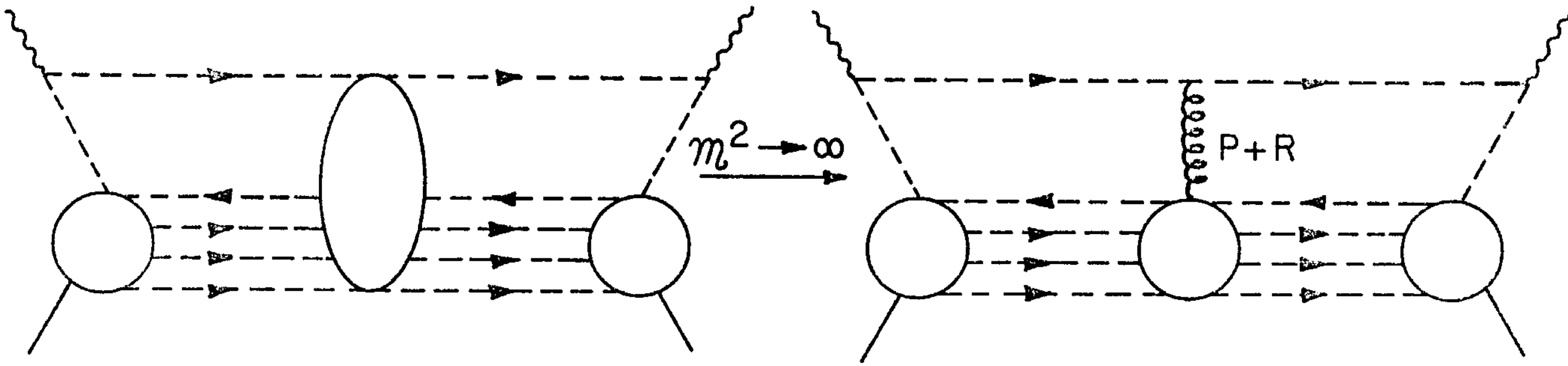


Fig. 12

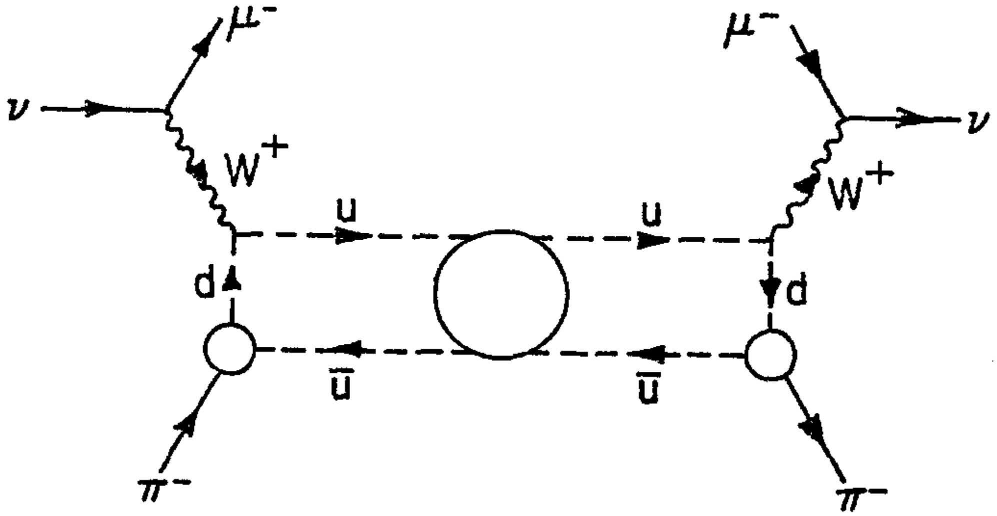


Fig. 13

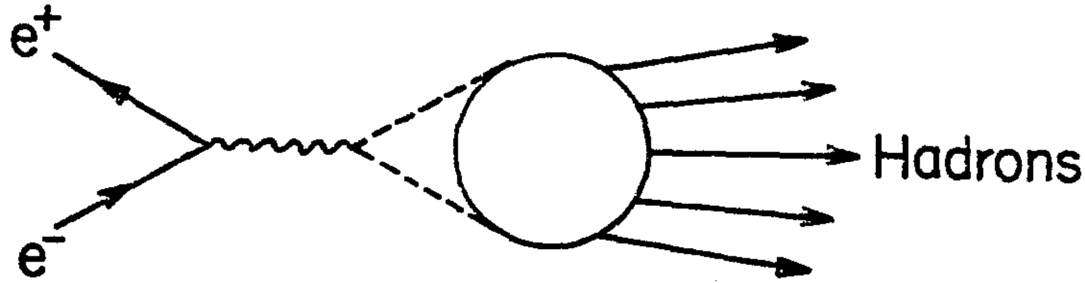


Fig. 14a

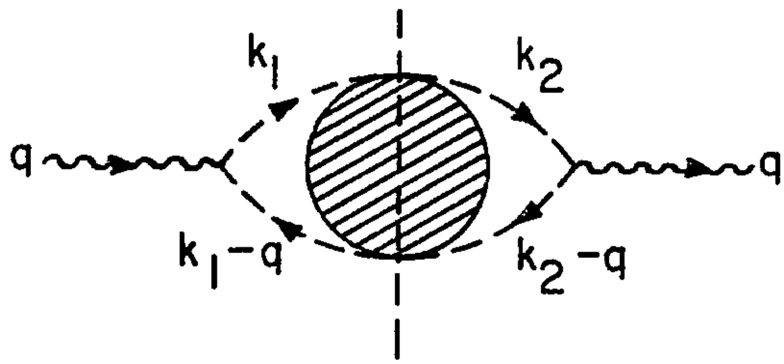


Fig. 14b

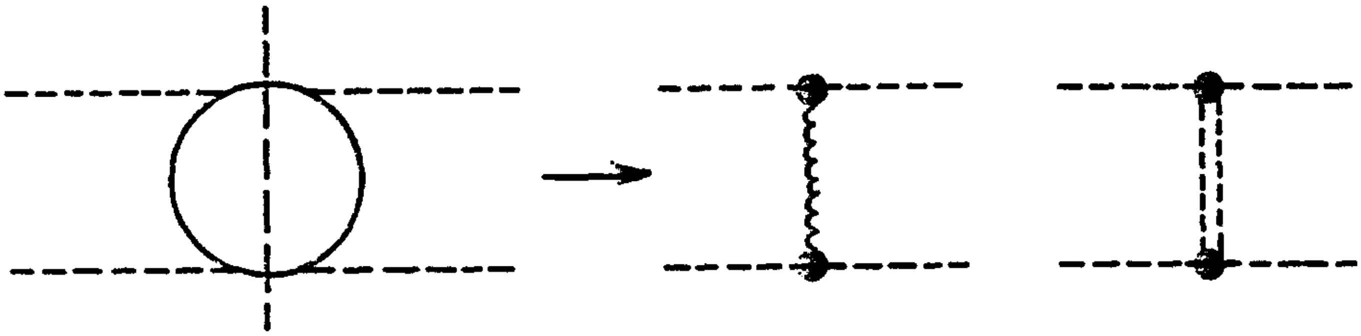
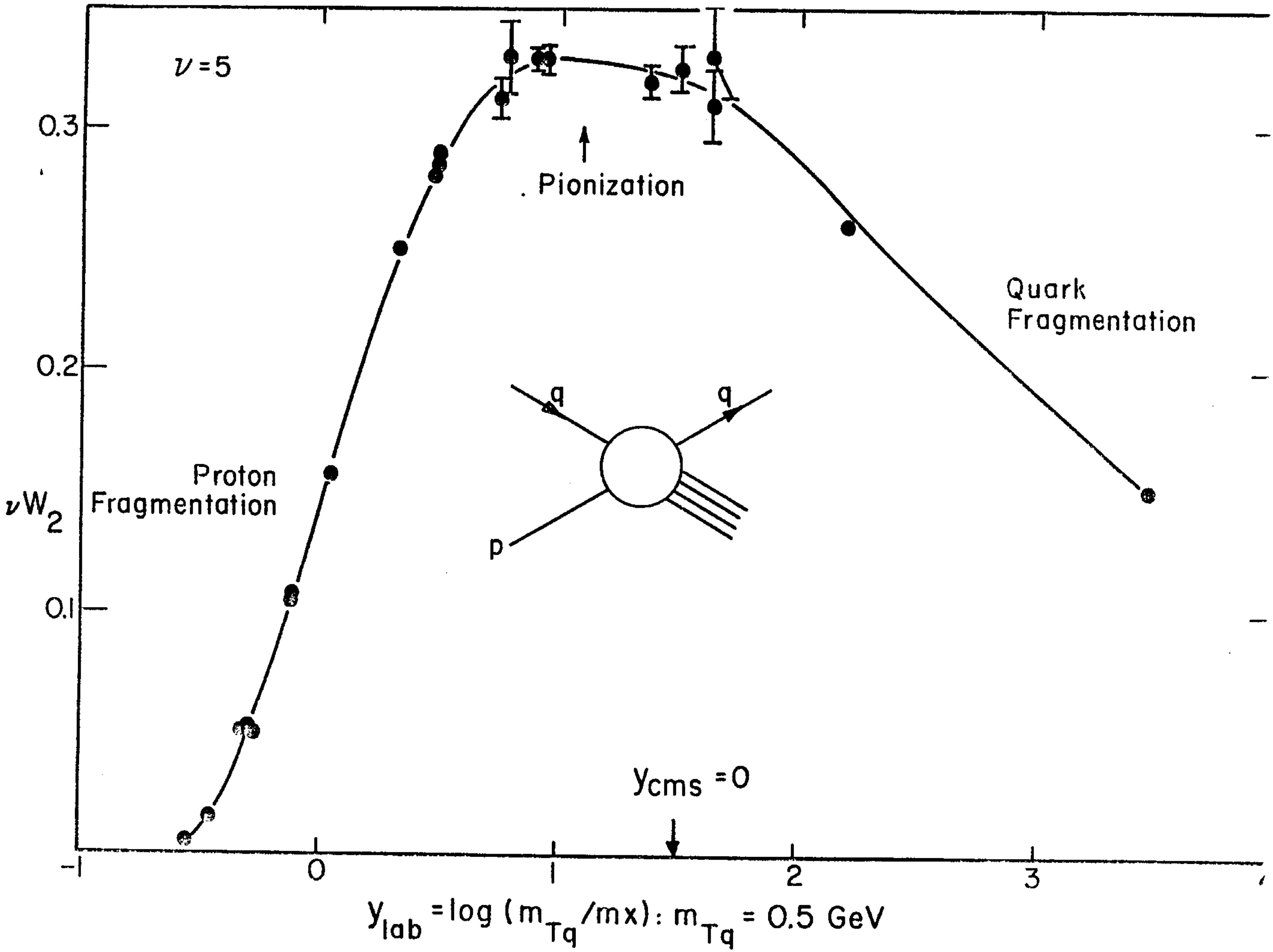


Fig. 15



ν=5
0.3
0.2
0.1
νW2
Pionization
Quark
Fragmentation
Proton
Fragmentation
q
q
p
ycms =0
-1
0
1
2
3
ylab =log (mTq/mx): mTq = 0.5 GeV

Fig. 16

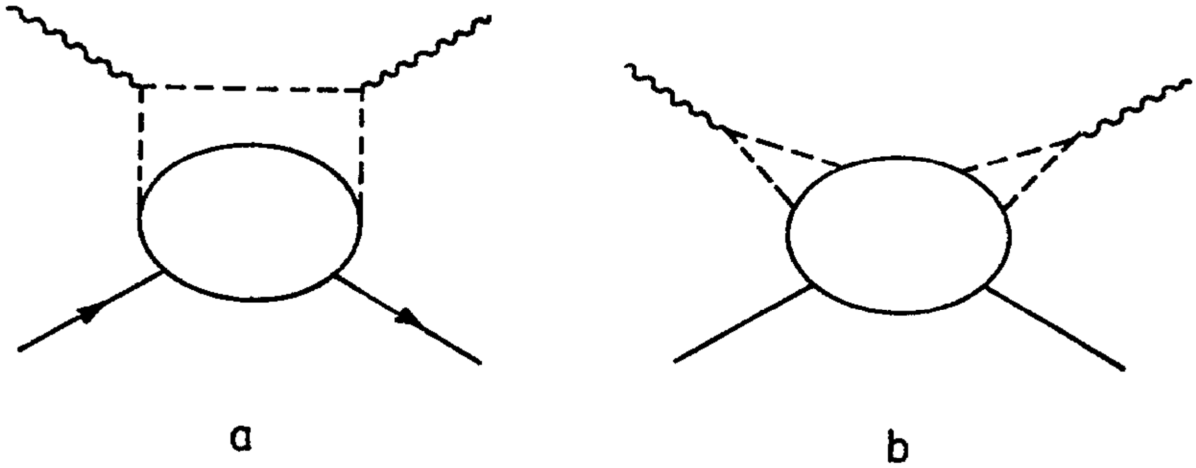


Fig. 17